INTERMEDIATE AERODYNAMICS

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To My Wife

FRANCES CREEF TRUITT

PREFACE

The aeronautical engineering student is faced with the task of correlating his previous knowledge of basic engineering courses into a more specific vein of thought. If the student is confronted with a text that has been written on elementary aeronautics, from a more or less general theme, he finds himself with a fair knowledge of nomenclature and only a descriptive view of aerodynamic phenomena. With this type of training, the engineering student is apt to get the idea that the course is either too little developed or the teacher is inadequate. In either case the student has been retarded and valuable time has been lost, never to be regained. On the other hand, if the student is embarked on his aeronautical training with a too advanced text he finds not only that the subject matter is over his head, but the important fundamental ideas have been deleted in favor of more classical and complicated phenomena. Here again, an undesirable situation arises that may cause discouragement and possible disinterest to the extent of abandonment for fields that do not put so great a burden on him.

This text has been written in an effort to solve both of these problems by including necessary descriptive material along with mathematical explanations in order to emphasize the practical employment of aerodynamic theory. It is the author's belief that, although the student will be equipped with a necessary working knowledge of mathematics through calculus, the practical problems lend themselves readily to relatively simple mathematical solutions.

The author would like, at this point, to disclaim originality in all but the arrangement and presentation of the subject matter. It has been the author's desire to collect and correlate to the best of his ability those facts and theories which go to make not only a more clear and accurate application of aerodynamics, but to make the student proud of his profession.

To the National Advisory Committee for Aeronautics, and especially to Mr. John F. Victory, Secretary of the Committee, who has been so kind as to make available many of the Committee's original drawings reproduced in this book, the author wishes to express his gratitude. Credit is given to the Committee as well as to other sources throughout the text.

Valuable aid in regard to the publication of the material was received from Dr. William G. Friedrich, Head of the Aeronautical

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CONTENTS

AERONAUTICAL SYMBOLS

W	Weight
g	Standard acceleration of gravity, 32.2 ft/sec^2
ρ (rho)	Density (mass per unit volume)
S	Area
b	Span
c	Chord
μ (mu)	Coefficient of viscosity
V	True airspeed
$\frac{\rho V^2}{2}$	Dynamic (impact) pressure $= q$
C.P.	Center of pressure
α	Angle of attack
T	Thrust
Q	Torque
η (eta)	Efficiency
n	Revolutions per sec (rps)
N	Revolutions per minute (rpm)
ϕ (phi)	Effective helix angle
β (beta)	Propeller blade angle

INTRODUCTION

Physical Laws

Although the science of aerodynamics is a relatively new one by name, it is founded on the laws of physics. Some new applications of these physical laws will be encountered and, in order to emphasize the more fundamental ones, a short introductory review is given.

Inertia. Newton's first law of motion states that a body in motion or at rest will tend to remain in motion or at rest unless acted on by some outside force. This tendency of a body is called *inertia.* The force with which a body offers resistance to change in its present state is called the *inertia force,* or *force of inertia.*

A force must be applied to change a body, either to put it in motion, or if in motion, to change its speed or direction. If a body experiences either of these changes, it is evident that a force has been applied to the body.

Mass is the *quantity* of a material. It is not the same as volume, because it is found that a given amount or *mass* of a gas may occupy one volume under one condition of pressure and temperature, while it will occupy an entirely different volume when the pressure and temperature are changed. Mass is not the same as weight because weight is the force with which a mass is pulled toward the center of the earth. This can probably be demonstrated best in the following formula:

$W = Mg$ where W = weight in pounds
M = mass in slugs
g = acceleration of gravity. (At 40° latitude is taken as approximately 32.2 ft/sec/sec)

then $$M = W/g$$

Example: If air weighs 0.07651 lb/cu ft at standard sea-level pressure of 14.7 lb/sq in. and temperature of 59° F, and the acceleration of gravity at standard conditions as referred to above, then

$$M = W/g$$

or, *mass* of air at standard conditions = 0.07651/32.2 = 0.002378 slug/cu ft.

It is especially important that the student take special note of the example because the expression 0.002378 slug/cu ft is standard conditions for air and is used in many computations.

We are familiar with the second of Newton's laws of motion, which says that the force required to produce a change in the velocity of a body is proportional to mass of the body and the rate of change in velocity. In the form of an equation:

$F = MA$ where F = force in pounds
M = mass in slugs
A = acceleration in ft/sec/sec

where *acceleration* is equal to a change in velocity per unit length of time.

Example: A plane weighing 5000 lb is preparing to take off and is traveling now at the rate of 20 mph. What force is required to bring its speed up to 50 mph: (a) in 5 sec? (b) in 10 sec? Neglect friction. (g = 32.2 ft/sec/sec.)
Solution: $M = W/g$ = 155.3 slugs (Change in velocity 50 mph − 20 mph = 30 mph so 30 × (44/30) = 44 ft/sec)

(a) Acceleration, a = 44 ft/sec in 5 sec = 8.8 ft/sec/sec. Force, $F = Ma$; 155.3 × 8.8 = 1367 lb
(b) Acceleration, a = 44 ft/sec in 10 sec = 4.4 ft/sec/sec. Force, F = 155.3 × 4.4 = 683 lb

Vectors. A measurement which has both direction and magnitude is a *vector.* Force, as referred to above, is always a vector quantity. In expressing the force, not only the magnitude but also the direction in which the force is acting must be stated. It may be said that a plane travels at a speed of 200 mph but this is not a vector quantity because no direction is given; however, when it is said that a plane moves northward at 200 mph, this can be represented as a vector because it denotes direction and velocity.

The term *resultant* is actually the result of two or more forces that act simultaneously on a body.

Example: A force of 8 lb is acting on a body toward the north and at the same moment a force of 6 lb is acting toward the east.
Find the resultant force (magnitude and direction). (See Fig. I-1.)
Solution: From A as the origin, plot the vector AB toward the north, 8 units long. Then from point A toward the east draw vector AC with magnitude of 6 units. Completing the parallelogram as indicated in Fig. I-1 and drawing the resultant AD we find its force as 10 units and direction 36° 52′.

Likewise, one may have a resultant force from which he is able to plot a number of components as shall be seen later. Another consideration is when two equal forces are acting opposite to each other. The resultant is, of course, equal to zero, or the body acted upon is said to be in equilibrium.

In the case of an airplane in level, unaccelerated flight, the four

fundamental forces acting on the plane can be represented by vector quantities. Two of these vectors are *lift* and *gravity*. It can be seen at once that, if the plane is flying level, the two components, lift and gravity, are acting opposite to each other and the resultant is

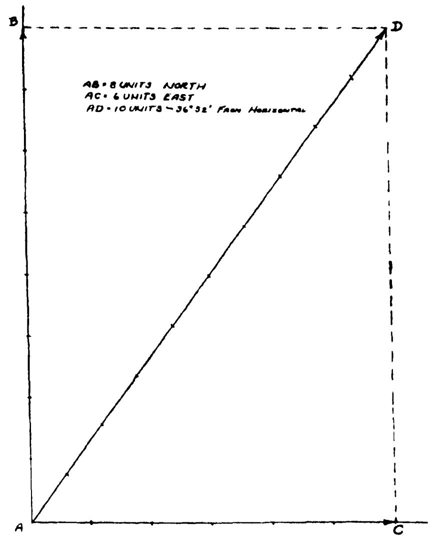


Fig. I-1. Resultant of two vectors.

equal to zero; the airplane is neither climbing nor diving. In the same manner, if the plane is flying unaccelerated, the other two forces, *thrust* (propulsion of the propeller) and *drag* (for every action there is an equal and opposite reaction), are equal and opposite to each other and the two forces are said to be in equilibrium. There-

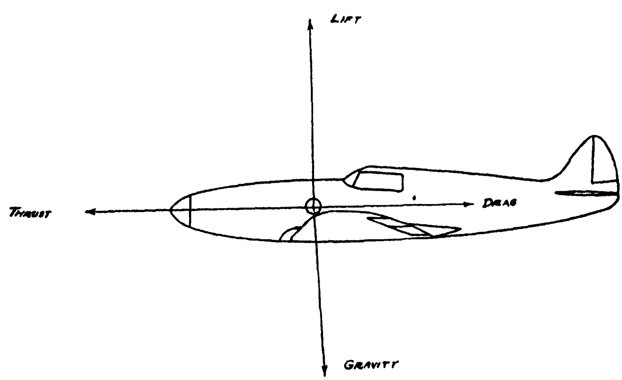


Fig. I-2. Equilibrium of forces on airplane.

fore, the resultant of forces acting on a plane in unaccelerated level flight are said to be equal to zero or in equilibrium. (See Fig. I-2.)

Moments. When a force is applied to a body and produces rotation around a given axis it is called the *moment* of the force with respect to the axis; the direction and amount of the *moment* depending upon the direction of the force and its distance from the axis. The moment is the product of the forces and the moment arm; the moment arm being the perpendicular distance from the axis to the point of applied force. For example, a force of 10 lb at a perpendicular distance of 2 ft from the axis produces a rotating moment of 20 ft-lb.

The convention of signs adopted in order to determine moments tending to produce rotation in opposite direction is from the N.A.C.A.; those moments tending to generate *clockwise* rotation are called *positive,* those moments generating in counterclockwise direction

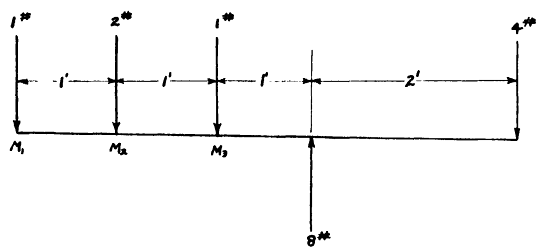


Fig. I-3. Moments in equilibrium.

negative. When the *positive moments* equal the *negative moments* (in other words, when there is no rotation) the algebraic sum of the moments is zero. This is expressed in the equation $\Sigma M = 0$ which means the sum of all the moments, M, both positive and negative are equal. Calculating the moments about a point A (Fig. I-3):

A: M_1 acts in counterclockwise direction at a distance of 3 ft and a force of 1 lb $= -3$ ft-lb moment.

$$M_2 = -4 \text{ ft-lb}; \; M_3 = -1 \text{ ft-lb}; \; M_4 = +8 \text{ ft-lb}$$

then $\quad M_1 + M_2 + M_3 + M_4 = -3 - 4 - 1 + 8 = 0$

or, using notation, $\Sigma M = 0.$

It is apparent that all the moments are in equilibrium about axis, A, hence there is no rotation.

With the above condition, there will be no rotation but unless the axis is supported by an 8-lb force upward opposite to the downward force of 8 lb there will be a downward movement. When the

total vertical forces equal zero there is neither rotation nor translation and the body is said to be in equilibrium.

Conventional N.A.C.A. signs denote *downward forces negative* and *upward forces positive.* For vertical equilibrium, the algebraic sum of the vertical forces is zero. This is expressed by $\Sigma V = 0$.

For a system of horizontal equilibrium the expression is $\Sigma H = 0$. By vector analysis, forces at an angle can be resolved into vertical and horizontal components and again the conditions for equilibrium are:

$$\Sigma H = 0, \Sigma V = 0 \text{ and } \Sigma M = 0$$

The above conditions for equilibrium can be applied to an airplane in motion because of the fact that the original motion is not affected in any way by this superimposed system.

Specific Gravity–Density. Specific gravity is the ratio of the weight of a solid or liquid to the weight of an equal volume of water at some standard temperature. The specific gravity of gases is usually referred to hydrogen as a base. The *density* of a body is mass per unit volume.

$$D = M/V$$

In the case of solids the ratio of mass to volume is not changed, which, of course, is not true in the case of gases.

Suppose a balloon is filled with air to a volume of 6 cu ft. A definite quantity of air is said to be confined in the balloon. If the air is heated and the balloon is expanded to 12 cu ft, the volume will be doubled but the quantity of air confined in the balloon will remain the same. The density of the air now, however, will be one-half what it was before heating. If, however, the temperature is held constant and the volume is decreased to 3 cu ft, the density will be doubled. A combination of the two laws of gases, *Boyle's law* and *Charles' law,* gives the familiar expression for a given kind of gas

$$PV/MT = R$$

where P = pressure
V = volume
M = mass
T = absolute temperature
R is a constant (depending on the gas)

Standard Atmosphere. The density of air varies with the temperature, pressure, etc. The standard (N.A.C.A.) conditions are a barometric pressure of 29.92 in. (or 76 cm) of mercury and temperature of 59° F (or 15° C). As was pointed out before, the *mass density* (ρ = rho) of air is 0.002378 slug per cu ft. Air conforms to Boyle's and Charles' laws in that the volume of a gas varies inversely with pressure and directly as the absolute temperature.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

where P_1, V_1, T_1 = pressure, volume and absolute temperature under standard conditions

P_2, V_2, T_2 = other than standard conditions

Weight density is the weight divided by volume. Therefore the density of air is increased by an increase in pressure, a decrease in volume, or a decrease in temperature. In a given volume, the density and specific weight of air vary directly as pressure and inversely with absolute temperature.

$$\frac{\rho_1}{\rho} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

where ρ (rho) = mass density of air at standard conditions

ρ_1 = mass density of air at some other condition

Example: What will be the density of dry air if the pressure is 22.34 in. of mercury and the temperature is 25° F?

The absolute temperature (T) = 25° + 459.4° = 484.4° F abs.

$$\rho_1 = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \times \rho = \frac{22.34}{29.92} \times \frac{518.4}{484.4} \times 0.002378 = 0.00190 \text{ slug/cu ft}$$

Example: What is the specific weight of dry air if the pressure is 15.12 in. of mercury and the temperature is −15° F?

$$\rho_1 = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \times \rho = \frac{15.12}{29.92} \times \frac{518.4}{444.4} \times 0.002378 = 0.00140 \text{ slug/cu ft}$$

$$\text{Specific weight} = \rho \times g = 0.00140 \times 32.2 = 0.0451 \text{ lb/cu ft}$$

PROBLEMS

1. What is the density of dry air if the pressure is 28 in. of mercury and the temperature is 60° F?
2. Find the specific weight of dry air if the pressure is 19 in. of mercury and the temperature is 0° F?
3. Find the density of dry air if the temperature is 20° C and the pressure is 30 in. of mercury.
4. What is the density of dry air if the temperature is −10° F and the pressure is 20 in. of mercury? What is the specific weight?

Atmosphere. It is essential for the student of aeronautics to be somewhat familiar with the atmosphere and its composition. The air is often referred to as a perfect fluid. Air is a gas. It is a physical mixture, not a compound.

The earth's atmosphere at sea level has the following percentages by volume of these gases: (a) Nitrogen—78.08; (b) Oxygen—20.94; (c) Argon—0.94; (d) Hydrogen—0.01; (e) Neon—0.0012; (f) Helium—0.0004; (g) Carbon Dioxide—0.03.

Water vapor is always present in the atmosphere, the amount

varying with the temperature and other factors, but averaging about 1.2% at the earth's surface.

Up to altitudes of 50,000 ft movements caused by high- and low-pressure areas and the rotation of the earth will be found. These currents tend to keep the air in a homogeneous mixture.

TABLE I

ALTITUDE-PRESSURE-DENSITY RELATION BASED ON N.A.C.A. NO. 218

Altitude (ft)	Temperature (° F)	Pressure (in Hg)	ρ	ρ/ρ_0	$\sqrt{\rho_0/\rho}$
0	59.0	29.92	0.002 378	1.000	1.000
1 000	55.4	28.86	0.002 309	0.9710	1.0148
2 000	51.8	27.82	0.002 242	0.9428	1.0299
3 000	48.4	26.81	0.002 176	0.9151	1.0454
4 000	44.8	25.84	0.002 112	0.8881	1.0611
5 000	41.2	24.89	0.002 049	0.8616	1.0773
6 000	37.6	23.98	0.001 987	0.8358	1.0938
7 000	34.0	23.09	0.001 928	0.8106	1.1107
8 000	30.6	22.22	0.001 869	0.7859	1.1280
9 000	27.0	21.38	0.001 812	0.7619	1.1456
10 000	23.4	20.58	0.001 756	0.7384	1.1637
11 000	19.8	19.79	0.001 701	0.7154	1.1822
12 000	16.2	19.03	0.001 648	0.6931	1.2012
13 000	12.6	18.29	0.001 596	0.6712	1.2206
14 000	9.2	17.57	0.001 545	0.6499	1.2404
15 000	5.5	16.88	0.001 496	0.6291	1.2608
20 000	−12.3	13.75	0.001 267	0.5327	1.3701
25 000	−30.1	11.10	0.001 065	0.4480	1.4940
30 000	−48.1	8.88	0.000 889	0.3740	1.6352
35 000	−65.8	7.04	0.000 736	0.3098	1.7961
40 000	−67.0	5.54	0.000 582	0.2447	2.0215
45 000	−67.0	4.36	0.000 459	0.1926	2.2786
50 000	−67.0	3.44	0.000 361	0.1517	2.5674

The air in the atmosphere near the earth's surface is compressed by the weight of the air above it. Hence, the higher the altitude the less pressure, because there is less air above it to cause pressure.

The lower region of the earth's atmosphere is called the *troposphere.* In the troposphere temperature decreases with altitude, winds may blow in any direction and, since moisture is present, clouds are found. The upper region of the earth's atmosphere is called the *stratosphere.* The stratosphere contains the so-called isothermal region where the temperature remains constant. In the troposphere the temperature decreases with altitude but it decreases to approximately −67° F in the stratosphere, and remains practically the same. In the stratosphere, there is no dust or alkali and no moisture is present. Without these condensation nuclei no clouds can be formed.

The dividing line between the troposphere and the stratosphere is called the *tropopause.* The altitude of the tropopause varies with

the regions. It is 56,000 ft in the Tropics, about 38,000 ft over the United States, and about 28,000 ft in the Polar Regions.

Effect of Humidity. Students of physics will be familiar with the terms *humidity* and *dew point.* Although the atmosphere practically always has some moisture present, the standard atmosphere which has been adopted is assumed to be perfectly dry.

Water vapor is lighter in weight per unit volume than air; under standard conditions (29.92 in. pressure and 59° F temperature) its density is approximately 0.001476 slug per cu ft. Its density ratio is 0.001476/0.002378 or approximately 5/8 of the density of air. The portion of the atmosphere that is water vapor weighs 3/8 less than if it were dry air. By the theory of partial pressures, the weight of a cubic foot of moist air equals the weight of dry air plus the weight of the water vapor.

1
THE AIRFOIL

Students of aeronautics sometimes overlook the fact that air has *weight* and *inertia*. The blanket of atmosphere around the earth weighs about 6 million, billion tons. On a cold day a cube of air one city block on each side weighs over 8 million lb. The weight of a given volume of air varies considerably with temperature and pressure, as discussed in the Introduction.

Because of its weight air possesses inertia, which causes it to resist a change in direction and also changes in speed. When a *mass* of air is made to change its speed or direction or both, an *acceleration* takes place and a *force* is evolved. A fast transport plane influences more than 125 tons of air each second when flying at 250 mph.

If a moving mass of air is considered to be a concentration of solid particles striking an inclined plane, the idea of *impact force* can be clearly understood. In reality, air consists of tiny particles (molecules) which create a reactionary force in the same manner as solid particles striking a surface. In addition to the impact force, the air which is a gaseous fluid, obeying laws of fluid flow, creates a force on the upper surface of an *airfoil*. An *airfoil* is any surface, such as an airplane *wing*, *aileron*, or *rudder*, designed to obtain a useful *reaction* or *force* from the air through which it moves.

The flat airfoil develops lift by imparting a downward momentum to the air. The sustentation is the result of inertia force on the lower surface, which force is created by deflecting air downward (for every action there is an equal and opposite reaction).

Cambered Airfoils are those which are curved in cross section as shown in Fig. 1-1. They are described by giving the *coordinates* of points along the upper and lower surfaces, the leading edge or the projection of the leading edge on the *chord line* being taken as the *origin*. These coordinates will be given in percentages of the chord length.

The three particular dimensions that are important are:

1. The position, x/c, of the maximum value of a/c, called the *position of the maximum ordinate*.
2. The maximum value of a/c, called the *amount of upper camber*.
3. The maximum value of b/c, called the *amount of lower camber*.
4. The maximum value of m/c, called the *camber of the mean line*.

In variations of these quantities and different combinations thereof, a basis has been made for a large amount of wind-tunnel research in an effort to obtain the most effective airfoil section for airplanes.

The early usual method of designing an airfoil was to draw a section that "looked good" and then test it in the *wind tunnel*, making subsequent modifications to improve a section that gave promising results in an earlier test.

This procedure has been practically eliminated by a series of tests by the N.A.C.A. at Langley Field, Virginia, which showed a systematic variation with thickness ratio of the section.

Figures 1-2, 1-3, 1-4 (RN 460, 537, 586, and 610) show the derivations of the profiles by changing systematically these shape variables.

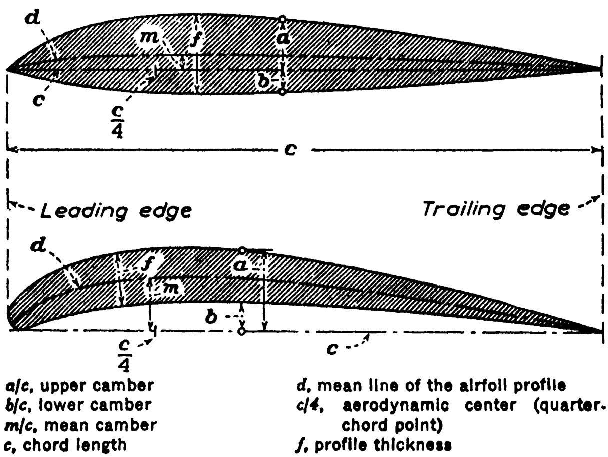


Fig. 1-1. Dimensions of an airfoil profile, N.A.C.A.

The symmetrical *profiles* were defined in terms of basic thickness variation, symmetrical *airfoils* of varying thickness being obtained by the application of factors to the basic ordinates. The cambered profiles were then developed by combining these thickness forms with various *mean lines*. These mean lines were obtained by varying the camber and the shape of the mean line to alter the position of the maximum mean-line ordinate. In the N.A.C.A. reports, the maximum ordinate of the mean line is referred to as the camber of the airfoil, and the position of the maximum ordinate of the mean line as the position of the camber. An airfoil, as described above, is usually designated by a number consisting of four digits:

The first indicates the *camber* (maximum ordinate of mean line) in percent of the chord;

The second, the *position* of the camber in tenths of the chord from the leading edge;

The third and fourth, the maximum *thickness* in percent of the chord.

For example: The N.A.C.A. airfoil 6512 has a maximum camber of 6% of the chord at the position 0.5 of the chord from the leading edge, and a maximum thickness of 12% of the chord; the N.A.C.A. 0009 airfoil is a symmetrical airfoil having a maximum thickness of 9% of the chord. The N.A.C.A. 24012 has a camber of approximately 2% of the chord (see Table I) at 0.2 of the chord from the leading edge and a thickness of 12% of the chord. Note that airfoils such as N.A.C.A. 2212 and the N.A.C.A. 24012 have practically the same

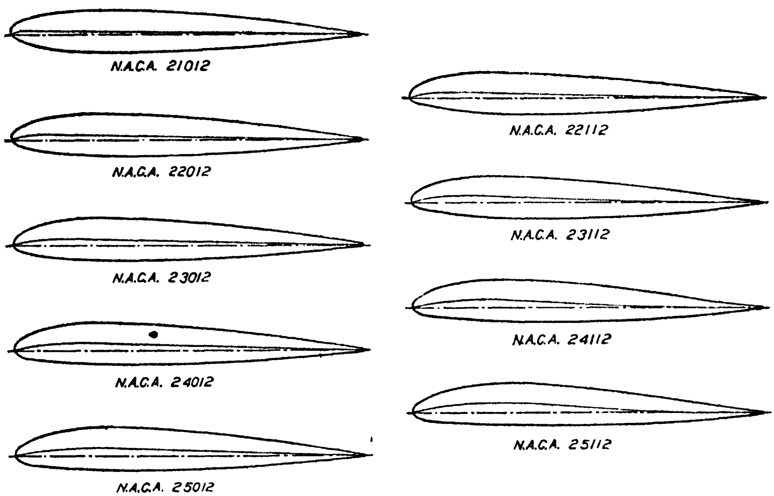


Fig. 1-2. Airfoil families with maximum camber unusually far forward.

camber, camber position, and thickness; however, the shapes of the mean camber lines, designated by the digit 2 in one case and 40 in the other, are entirely different.

The airfoils designated by the four- and five-digit numbers have only one form of thickness variation. Changes in the form of the thickness variation made by altering the leading-edge radius and the position of the maximum thickness have been designated by appending two additional digits (see N.A.C.A. Technical Report No. 610) and are separated by a dash from the basic airfoil designation. The first of these two digits indicates the relative magnitude of the leading-edge radius and the second indicates the position of the maximum thickness in tenths of the chord from the leading edge. The significance of the leading-edge radius designation is given below:

0 designates sharp leading edge,
3 designates ¹/₄-normal leading-edge radius,
6 designates normal leading-edge radius,
9 designates three or more times normal leading-edge radius.

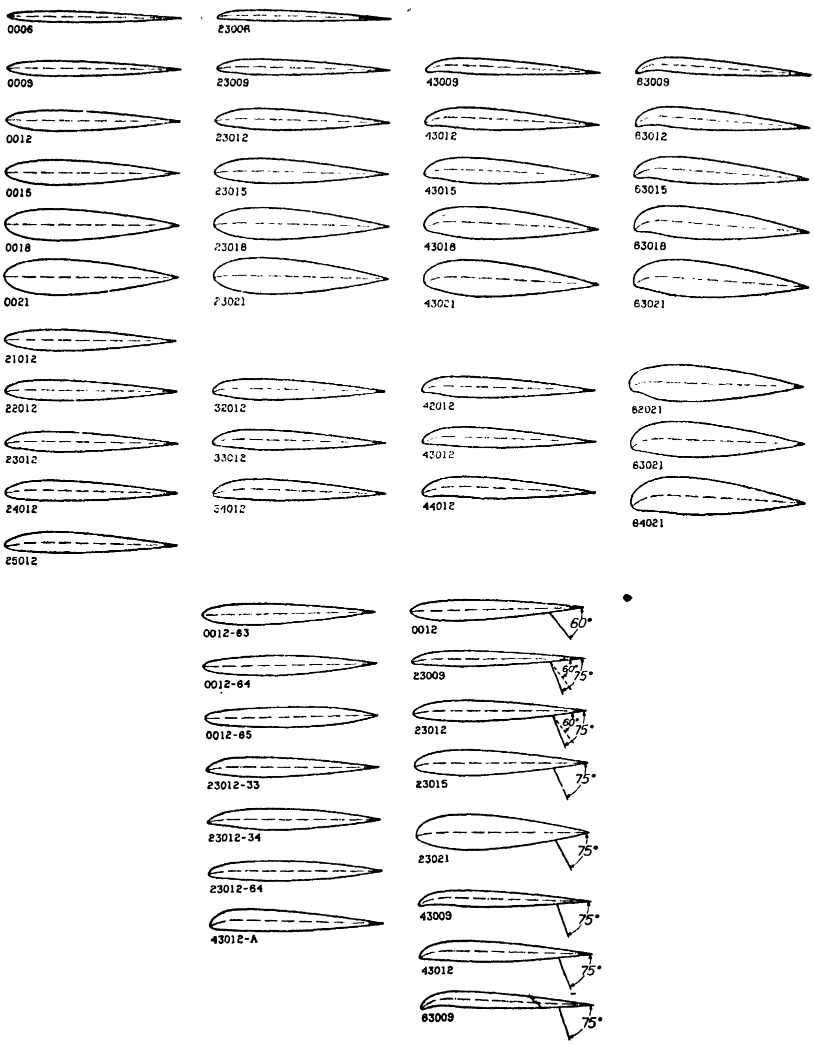


Fig. 1-3a. Airfoil profiles.

That is, the N.A.C.A. 0012–64 is a symmetrical airfoil having a normal leading-edge radius and the maximum thickness at 0.4 of the chord from the leading edge. The N.A.C.A. 24012-33 has the same mean line and thickness as the N.A.C.A. 24012 but has a leading-edge

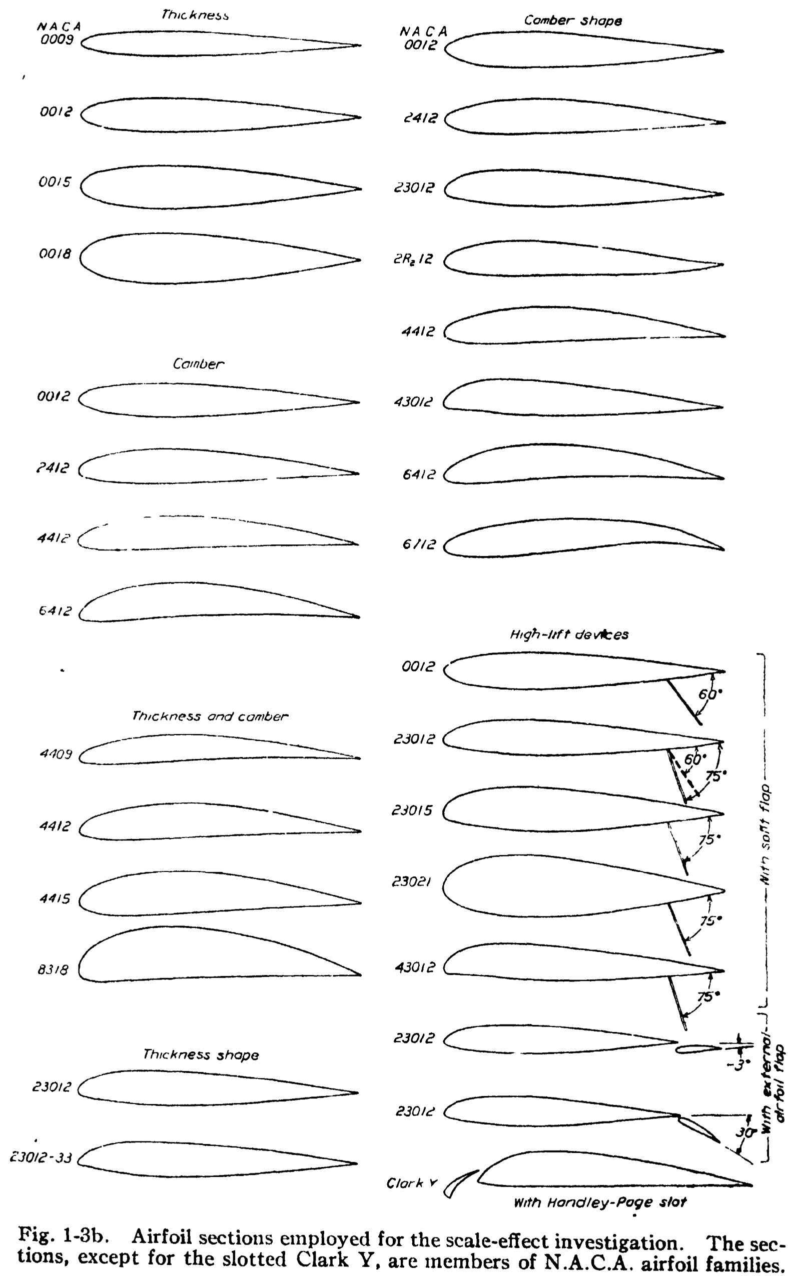


Fig. 1-3b. Airfoil sections employed for the scale-effect investigation. The sections, except for the slotted Clark Y, are members of N.A.C.A. airfoil families.

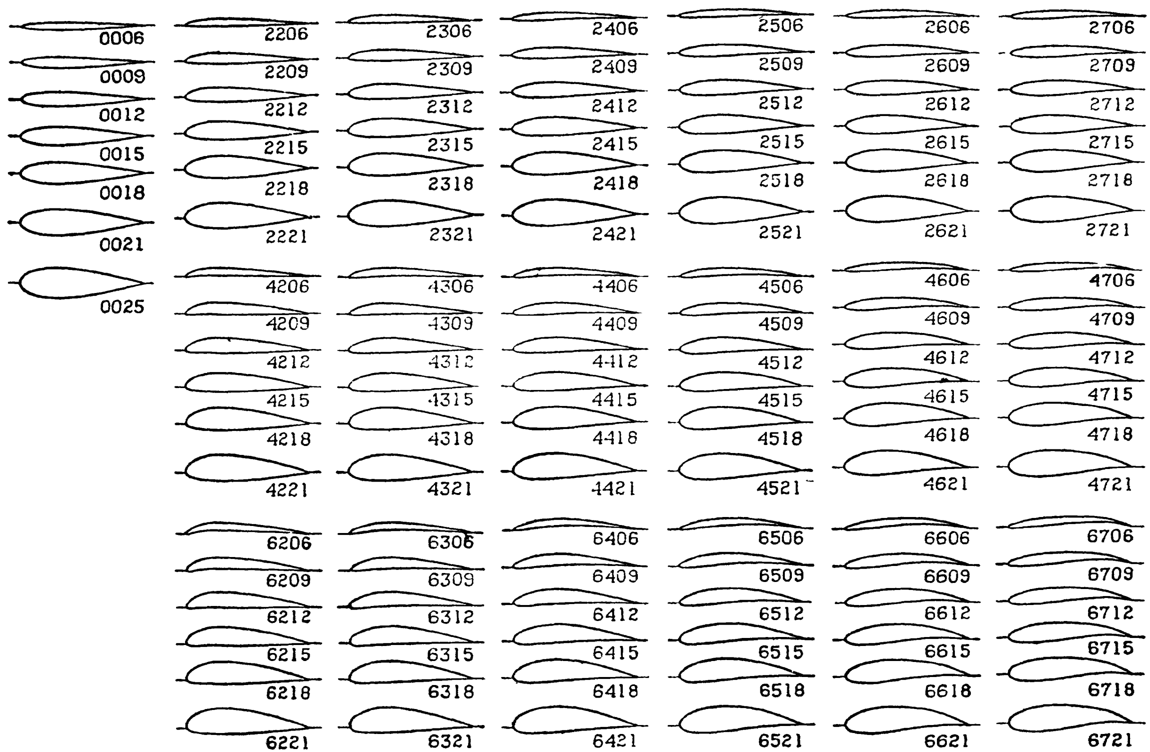


Fig 1-4 N A C.A airfoil profiles

radius $^1/_4$ the normal and the maximum thickness at 0.3 of the chord from the leading edge. Figure 1-2 shows a family of related airfoils that have the position of maximum camber unusually far forward. These airfoils gave unusually improved characteristics over those previously investigated, especially in regard to the pitching moment that will be discussed in a later chapter.

The investigation of a family of related airfoils, in N.A.C.A. Technical Report No. 460 (Fig. 1-2), indicated that the effects of camber in relation to maximum lift coefficients were more pronounced with airfoils having the maximum camber forward or back of the usual position of $^3/_{10}$ to $^5/_{10}$ of the chord. However, no attempt was made to test airfoils having extreme camber positions, because of the adverse pitching moment that undoubtedly could be expected.

A recent investigation of related airfoils, Fig. 1-4, indicated that if the position of maximum camber was made forward of the usual location, an increase of the maximum lift coefficient resulted. This series of forward-camber airfoils, which have been developed, shows airfoil characteristics superior to those tested previously.

With this type of report, in the useful range of shapes, a designer may select the best possible airfoil-section, and may predict to a reasonable degree the aerodynamic characteristics to be expected in flight.

Reynolds Number and Dimensional Relations. The only dimensions which appear in any branch of dynamics are *Mass* (M), *Length* (L), and *Time* (T). At this time, it seems appropriate to introduce an expression that includes these important dimensions. Expressing a factor of the resistance encountered by a solid moving through a real fluid:

$$\text{Reynolds Number } (RN) = f \text{ (function of) } (V, \rho, \mu, L)$$

where V = the speed of the relative motion between the solid and the fluid
ρ (rho) = the density of the fluid
μ (mu) = the coefficient of viscosity of the fluid
L = some linear dimension of the streamlined body

Speed of the relative motion may be expressed in terms of dimensions:

$$V = \frac{d}{t} = \frac{\text{distance}}{\text{time}} = \frac{L}{T};\ \rho = \text{density} = \frac{\text{mass}}{\text{volume}} = \frac{M}{L^3};$$

$$\mu = \text{coefficient of viscosity} = \frac{\text{slug}}{\text{ft-sec}} = \frac{M}{LT};\ L = \text{length of body} = L$$

Evaluating these in the following way we have:

$$RN = f\left(\frac{L}{T}, \frac{M}{L^3}, \frac{M}{LT}, L\right)$$

Supplying each of these factors with an exponent we have:

$$V^x = \left(\frac{L}{T}\right)^x;\ \rho^y = \left(\frac{M}{L^3}\right)^y;\ \mu^z = \left(\frac{M}{LT}\right)^z;\ L^{-1} = L^{-1}$$

We have then:

$$RN = f\left[\left(\frac{L}{T}\right)^x\left(\frac{M}{L^3}\right)^y\left(\frac{M}{LT}\right)^z L^{-1}\right]$$

Collecting exponents for each:

$$\begin{aligned} L&:\ x - 3y - z - 1 = 0 \\ M&:\ y + z = 0 \\ T&:\ -x - z = 0 \end{aligned}$$

then: $x = -z;\ y = -z.$ Therefore: $x = y.$

Substituting these values in the expression $x - 3y - z - 1 = 0$, we find

$$x = -1, \text{ then } y = -1 \text{ and } z = 1.$$

Then substituting these values in their proper places:

$$\left[\left(\frac{L}{T}\right)^{-1}\left(\frac{M}{L^3}\right)^{-1}\left(\frac{M}{LT}\right)^1 L^{-1}\right]$$

Therefore we may say:

$$(V^{-1}\rho^{-1}\mu^1L^{-1})$$

$$RN \text{ (Reynolds Number)} = \mu/\rho VL$$

The N.A.C.A. uses a value of 3.73×10^{-7} for coefficient of viscosity of air at 15° C. Except for extreme high and low pressure the coefficient of viscosity, μ, is independent of pressure.

The kinematic viscosity, ν, is the ratio of the coefficient of viscosity, μ, to the density of air.

$$\nu(\text{nu}) = \mu/\rho$$

Substituting this value in the above expression for Reynolds Number, we find a more familiar expression,

$$RN \text{ (Reynolds Number)} = \frac{\mu}{\rho VL} = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$$

Osburne Reynolds, an English physicist, made an extensive study of flow in pipes. He found that at low speeds the flow was smooth, but at high speed the flow was turbulent. He found experimentally that for values of RN (Reynolds Number) less than what is called the critical RN, the flow was laminar or smooth; for greater than the critical RN, the flow was turbulent.

Forces on Bodies. Consider now the study of forces evolved from a body of air moving with a velocity, V, striking a solid body, S, Fig. 1-5.

If a moving stream of air is caused to change speed, a force is required. If a stream of air is S sq ft in cross-sectional area, and is moving at a velocity of V ft/sec, then, at any point, P, a body of air, SV cu ft, is moving past point P, every second. As mentioned, if ρ (rho) is the mass of air in slugs per cu ft, a total mass of ρSV slugs is moving past the point, P, every second. If, in some way, the velocity is changed to another velocity in the same direction, a force is required to do this work. If this stream of air strikes against a wall so as to lose all of its velocity, by definition, it may be called acceleration (change of velocity per unit length of time). In other words, acceleration is, in unit time,

$$a = \frac{V - V_F}{t} \qquad \text{where } \begin{aligned} V &= \text{initial velocity} \\ V_F &= \text{final velocity} \\ &\quad (V_F = 0) \\ t &= \text{unit time} \end{aligned}$$

therefore $$a = V$$

The wall would exert a force (F = mass × acceleration) of $-\rho SV^2$, the minus sign denoting a force opposite in direction to the initial velocity, V. Actually, this theoretical value for force is not entirely true because of some disturbance at the face of the plate, due to the pocketing of the air before it is allowed to escape. Wind-tunnel experiments show that it is necessary to multiply the result by a correction factor of, roughly, 0.64. Then to express the pressure on a flat plate normal to the wind:

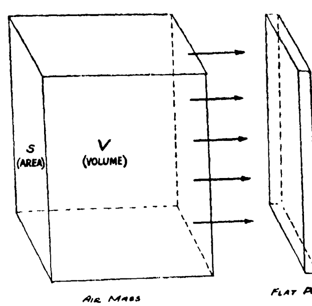


Fig. 1-5. Force of a body of moving air striking a flat plate.

$$P_n \text{ (pressure in lb)} = 0.64\, \rho S V^2$$

$$\text{where } \begin{aligned} \rho &= \text{slugs per cu ft} \\ S &= \text{area sq ft} \\ V &= \text{velocity in ft/sec} \end{aligned}$$

But, by definition of the kinetic energy of a cu ft of air moving at a velocity, V ft per sec,

$$KE = \frac{MV^2}{2} = \frac{\rho V^2}{2}$$

We find the usual expression for force or pressure:

$$P = 1.28\ \rho/2\ SV^2$$

In this expression it is found that for airspeed in mph it becomes

$$P = 1.28 \times \frac{0.002378}{2} \times S \times \left(V \times \frac{44}{30}\right)^2$$

where ρ = 0.002378 slug/cu ft at standard conditions

Example: A 35-mph wind is blowing against a plate-glass window 5 ft by 7 ft. If atmosphere is normal density what will be the force exerted on the window?

Solution:

$$\begin{aligned} P &= 0.00327\ SV^2 \\ &= 0.00327 \times 5 \times 7 \times (35)^2 \\ &= 140 \text{ lb} \end{aligned}$$

Example: If the 2 ft $\times$ 3 ft door of an automobile is suddenly opened, normal to the direction of movement, what will be the force in pounds on the door if the automobile is traveling at 85 ft per sec?

Solution:

$$\begin{aligned} F &= 0.00327\ SV^2 \\ &= 0.00327 \times 2 \times 3 \times \left(85 \times \frac{30}{44}\right)^2 \\ &= 113.7 \text{ lb} \end{aligned}$$

PROBLEMS

1. Find the total force in pounds of a 50-mph wind on a signboard 12 ft by 24 ft.
2. What force is required to move a 1 ft by 2 ft flat plate at a speed of 30 mph in a direction perpendicular to its surface?
3. What force is required to pull an airplane (with an equivalent flat plate area, E.F.P.A., of 4 sq ft) through the air at 120 ft per sec?

Bernoulli's Theorem of fluid flow states that when a fluid flows steadily through a pipe of varying cross section, the total energy (the ability to do work) remains constant. Considering the interior of the pipe as frictionless, energy may be found in three forms:

The energy due to static pressure, p;
The energy due to height or position, wz; and
The energy due to motion (kinetic), $wV^2/2g$.

Bernoulli's theorem for incompressible fluids in motion in a pipe is that a given weight (W lb) of fluid, with a specific weight of w lb per cu ft, moving with a velocity, V ft per sec, at a height z ft above some horizontal reference plane, and under a pressure of p lb per sq ft, has a constant energy value. Stated mathematically:

$$\frac{WV^2}{2g} + \frac{Wp}{w} + Wz = \text{a constant value}$$

The term $WV^2/2g$ may be written as $MV^2/2$, which is the familiar expression of kinetic energy of the moving mass; the second term Wp/w represents the pressure energy; and the third term, Wz, the potential energy. Dividing each term by W and multiplying each by w, the equation becomes

$$\frac{wV^2}{2g} + p + wz = \text{another constant}$$

If the fluid is air, the symbol ρ may be used for the mass density. The symbols V and p are the velocity and pressure, respectively, at any point in the pipe, and the air is considered as incompressible. Then for horizontal flow (z = height = constant):

$$\frac{\rho V_1^2}{2} + p_1 = \frac{\rho V_2^2}{2} + p_2$$

where V = velocity in ft/sec
p = pressure in lb/sq ft

Example: A horizontal water pipe is reduced in size from 24 in. diameter at a point A to 12 in. diameter at B. The flow in the pipe is 20 cu ft/sec, and the pressure at A is 20 lb per sq in. If it is assumed there is no loss in energy due to friction, what is the pressure at B?

Solution:

$$V_1 \text{ (at } A) = \frac{20}{\pi(1)^2} = 6.35 \text{ ft/sec}$$

$$V_2 \text{ (at } B) = \frac{20}{\pi(1/2)^2} = 25.5 \text{ ft/sec}$$

$$p_1 \text{ (at } A) = 20 \times 144 = 2880 \text{ lb/sq ft}$$

Then substituting in the formula $\dfrac{wV_1^2}{2g} + p_1 = \dfrac{wV_2^2}{2g} + p_2$

$$\frac{62.4 \times (6.35)^2}{64.4} + 2880 = \frac{62.4 \times (25.5)^2}{64.4} + p_2$$

$$p_2 = \frac{62.4 \times (6.35)^2}{64.4} + 2880 - \frac{62.4 \times (25.5)^2}{64.4}$$

$$p_2 \text{ (pressure at } B) = 15.9 \text{ lb per sq in.}$$

Therefore the pressure at B is less than A, since the fluid is restricted to a smaller cross section at that point. The above example may well be applied to any fluid so far as the increase of velocity and decrease of pressure is concerned.

Bernoulli's theorem may be written in another form:

$$p = \text{constant} - \frac{\rho}{2} V^2$$

The constant is considered as large and positive. With the above expression, the theorem states that:

Where the velocity is *high*, the pressure is *low*.

Where the velocity is *low*, the pressure is *high*.

PROBLEMS

1. The flow in a water pipe where the potential energy, due to height, is constant, is 10 cu ft/sec. If the pressure is 20 lb/sq in. where the diameter is 18 in., what will be the pressure at a point where the diameter is 6 in.?

2. If air flows through a pipe (horizontally) at the rate of 1000 cu ft/sec and the pressure is 28 lb/sq in. where the diameter is 3 ft, what is the pressure where the diameter is 2 ft?

3. If the pressure energy is constant while water is flowing through a pipe that is 1 ft in diameter at point A and the flow in the pipe is 25 cu ft/sec, what is the pressure at B if the diameter is 6 in.? (The pressure at A is 25 lb/sq in.)

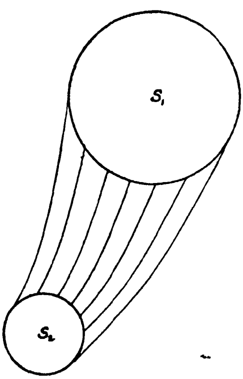


Fig. 1-6. Streamtube.

Streamlines are those lines the tangent to which, at any point, is the direction of the velocity at that point. A steady stream of air may be conceived as consisting of a number of particles moving in the same direction. The path of these particles is called a streamline. In steady flow, the shapes of the streamlines do not change because they are independent of time. If the shapes of the streamlines are continually changing, the flow is not laminar.

It would be impossible for two streamlines to cross because that would mean that two particles occupy the same space at the same time.

If the flow of any one streamline is such that the flow is in one plane and all other individual streamlines are entirely in that same place, or in a parallel plane, the flow is considered *two dimensional*. For this particular type of airflow, the pattern of the streamline shapes is identical in the entire series of parallel planes.

A streamline must be infinite in length, *i.e.*, it must be continuous or must form a closed path. Figure 1-6 shows a streamtube of air where the cross-sectional area, S_1, is shown in an enclosed path. The smaller cross section, S_2, is joined with the former by streamlines that are parallel to each other and in the same plane. Directly applying Bernoulli's theory to this introduces the idea of conservation of energy in that, regardless of varying the cross-sectional area, the energy value is constant at any point.

Streamline Flow over Airfoils. With Bernoulli's theory in mind, it is interesting to examine Fig. 1-7. The free air velocity, V_0, is taken to be normal density. In the various smoke tunnels, photographs have been taken which have shown the figure referred to as conforming closely to actual conditions as far as the streamlines are concerned.

It has been proved theoretically that, if the streamlines are shown closer together, the velocity is greater and the pressure less. If, from the free air position at *A* and *A'*, the pressure is normal and the streamlines have a definite spacing then, wherever any separation occurs, there would be assumed a decrease of velocity and increase of pressure. Applying Bernoulli's theorem in Fig. 1-7

$$\frac{\rho V_0^2}{2} + p_0 = \frac{\rho V_1^2}{2} + p_1$$

then

$$p_1 = p_0 - \frac{\rho}{2}(V_1^2 - V_0^2)$$

also

$$\frac{\rho V_0^2}{2} + p_0 = \frac{\rho V_2^2}{2} + p_2$$

$$p_2 = p_0 + \frac{\rho}{2}(V_0^2 - V_2^2)$$

$$p_2 - p_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$$

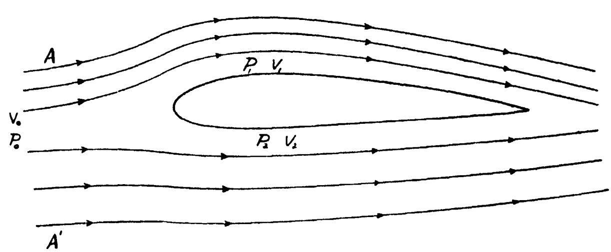


Fig. 1-7. Streamline flow over an airfoil.

It is evident that with the pressure at V_0 being atmospheric, the pressure, p_1, is less than atmospheric and the pressure, p_2, is greater than atmospheric. The difference between the pressure on the lower camber and pressure on the upper camber ($p_2 - p_1$) depends on the difference in the airspeeds over the top and bottom of the airfoil.

Hydrodynamic Theory of Lift. In explanation of Bernoulli's theory, it has been clearly demonstrated that there exists on the upper camber of the airfoil a low-pressure area, and a relatively high-pressure area on the lower camber.

Meteorological studies have established that high- and low-pressure areas exist over the earth and that the presence of a low-pressure area adjacent to a high-pressure area induces a circulation of the air from high to the low.

Many smoke tunnel tests show that there exist on the trailing edge of the airfoil small *vortices* which break off and remain in the air as the

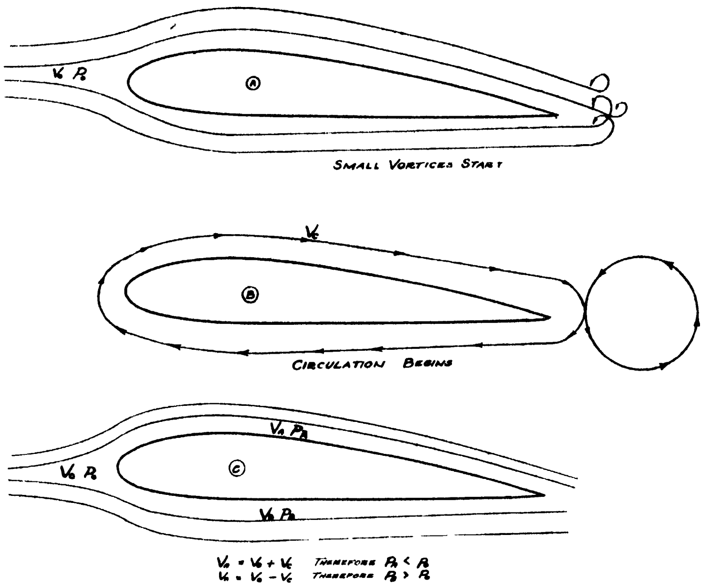


Fig. 1-8. Circulatory flow.

airfoil passes (see Fig. 1-8a). The trailing edge vortices act as a gear to set up a counter-circulation around the airfoil (see Fig. 1-8b), and these vortices increase in strength until the flow off the trailing edge is smooth. This action is set up very rapidly after the airfoil begins to move through the relative wind. The complete mathematical expression lies in the assumption that such a circulation gives the condition satisfying Bernoulli's theorem, in that the circulatory velocity, V_c, is subtracted from the velocity on the lower camber and added to the velocity over the upper camber.

The Centrifugal Force Idea of Lift. Figure 1-9 shows that there is

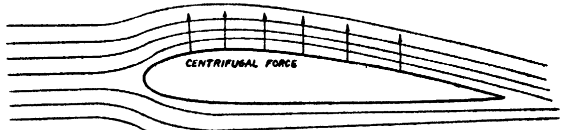


Fig. 1-9. Centrifugal force idea of lift.

definitely a change in the direction of the air over the upper and lower camber of the airfoil section. Actually the path of the air over the upper surface is curved and is deflected downward. The curved path of the air over the upper camber gives, at once, the idea of centrifugal force which tends to sling the air away from the surface. This action is sometimes thought of as causing a vacuum on the upper camber, and actually the airfoil has been found experimentally to produce 65% or more of its total lift from the upper camber.

Other theories embrace the idea that, as the airflow over the upper camber moves toward the trailing edge, there is imparted to the air a downward acceleration, *i.e.*, the air leaving the trailing edge has a component of velocity that is vertically downward in direction. The airfoil imparts a downward momentum to the air through which it passes and, as a result, a lifting force is created (for every action there is an equal and opposite reaction).

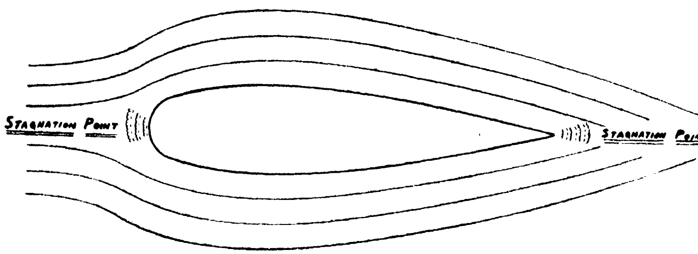


Fig. 1-10. Stagnation point.

Air-Particle Theory. This can best be demonstrated by the idea of two particles of air, A and B, at the *leading edge* of an airfoil. Considering these two particles relatively vertical to each other, the airfoil is pulled through these particles and particle A is given the path over the top of the airfoil, while particle B travels the underside. In order to conserve the idea of relativity between the fluid and body, it must be true that, at the trailing edge of the airfoil, these two particles must meet again at their original position. It is evident that it was necessary for particle A to travel the greater distance in the same length of time. Therefore its velocity must have been greater. This idea coincides nicely with Bernoulli's theory which states that "the higher the velocity the less the pressure."

Boundary Layer. If air flows with a velocity V around a stationary body, there is immediately adjacent to the surface a layer of air which has zero velocity. This is sometimes referred to as the "stagnation point." (See Fig. 1-10.) A short distance away, however, the air has its full velocity of V. Now consider a portion of the body

at a point ΔS, Fig. 1-11,* with a mass of air with velocity, V, acting at this point. As has been seen earlier, there must be a reaction such as force ΔF. This force can be resolved into two other forces ΔF_{tan} and ΔF_{nor}, the tangential and normal forces, respectively. It must be kept in mind that the ΔS is infinitely small and the area ΔS is considered as a flat, smooth surface. By definition of stress (force per unit area), $\Delta F_{\text{tan}}/\Delta S$ is the tangential stress, better known as the *shearing stress* between two layers of air. This shearing stress is also referred to as *skin friction*. The normal stress at the surface would be $\Delta F_{\text{nor}}/\Delta S$ and since the air is considered a perfect fluid the shearing stress is considered zero. The zone in which the velocity varies from zero to V is called the boundary layer. In the main airflow at a short distance from the surface, all of the particles have practically the same velocity and therefore there is little or no shearing stress between lay-

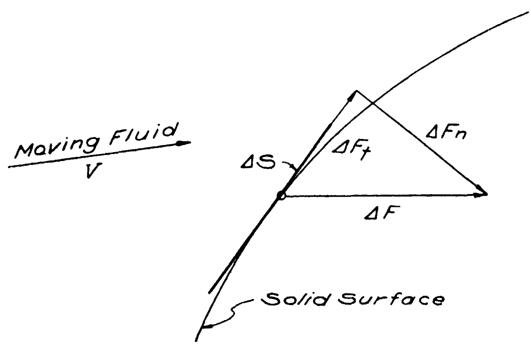


Fig. 1-11. Force components on a surface element.

ers of air. The viscosity of the air, then, would have no bearing and in aerodynamics air may be treated as a perfect fluid, *i.e.*, one having no viscosity. In the boundary layer, with its differences in velocity, viscosity is important in measuring the turbulence of the fluid flow. Here the value of the critical Reynolds Number carries such great weight in determining whether or not the flow is laminar or turbulent.

Scale Effect. Referring to an earlier discussion, Reynolds Number is

$$RN = VL/\nu$$

where V = the relative velocity between the body and the fluid
L = the length in ft of the body, usually the chord of the airfoil
ν = the kinematic viscosity of the fluid

* Reprinted by permission from *Aerodynamics of the Airplane* by C. B. Millikan, published by John Wiley and Sons, Inc.

In the study of aerodynamics the RN is important in that the relation of VL must be the same to compare the flow of air at different speeds around objects of varying size. A study of Fig. 1-12 will bring out clearly the idea of *geometrically similar flow.* The flow of air around object 1-12a would change to a flow like that of 1-12b if the velocity of air was increased. If the velocity of flow were the same as 1-12a and the size of the object were increased, the flow would be as shown in 1-12c. By decreasing the velocity in an inverse ratio to the increase in size, in other words, keeping the VL the same, the flow would be as shown in 1-12d. Keeping the VL the same, the flows are said to be geometrically similar. The expression (RN $= \rho VL/\mu$) is now so familiar that it will be understood that, with the size of the object in Fig. 1-12a and speed of flow were the same, if the density of the air increased, the flow would be changed to a flow resembling that in Fig. 1-12b, *i.e.*, the numerator ρVL of $\rho VL/\mu$ would be increased. Also a decrease in the *coefficient of viscosity* (μ) would have the same effect on the shape of the flow as an increase in velocity or density.

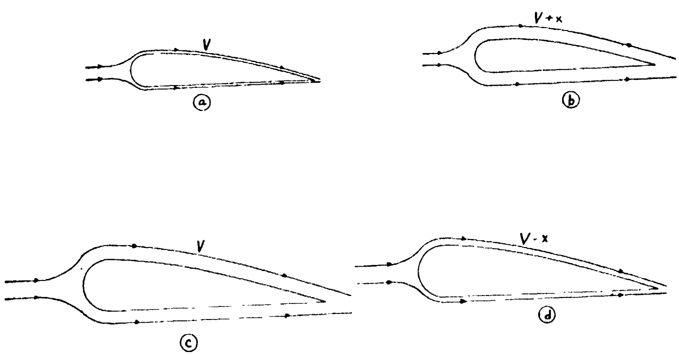


Fig. 1-12. Geometrical similar flow.

This is a way of saying that if the RN is the same, then the flows will be geometrically similar. If two bodies of different sizes have the same geometrical flow at corresponding points in the two flows the shape of flow and the direction of the streamlines will be the same and the forces generated will have the same ratio to each other.

The idea of geometrically similar flow is most important in airplane design because the use of the wind tunnel is essential. Models of wings or complete model airplanes may be tested in the wind tunnel, and the results of such tests may be used in the computation of the performance of the full-size airplane, provided the RN of the model is

the same as the RN of the airplane. The costs of testing full-size wing models would be prohibitive.

Since the models are small in size, compared with actual wings, in order that the RN be of the same magnitude as the RN of the actual wing, it is evident that the velocity or the density of the air must be increased in the same ratio of the decrease in size.

Example: It is proposed to find the probable lift and drag of an airfoil whose span is 100 ft and whose chord is 5 ft. The cruising speed is to be 100 ft/sec and is to be operated at sea level.

Solution: In computing the RN, velocity (V) must be in ft/sec; a linear dimension of the object (L) must be in feet (for wings the

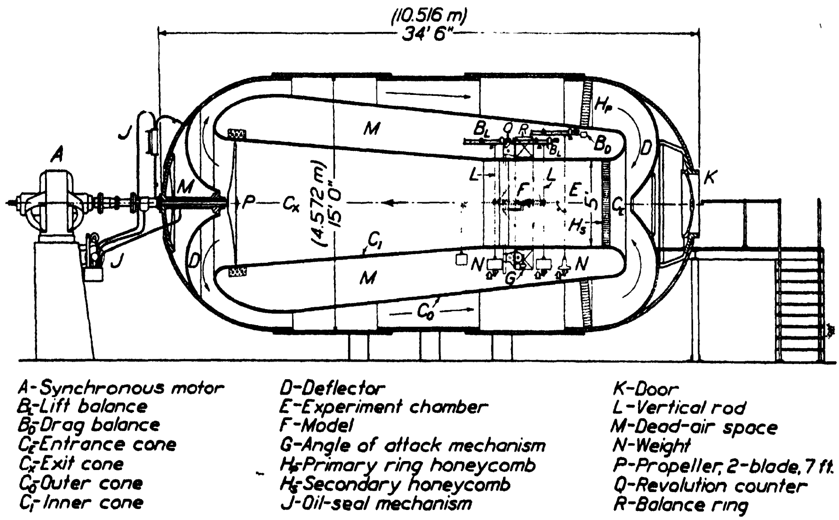
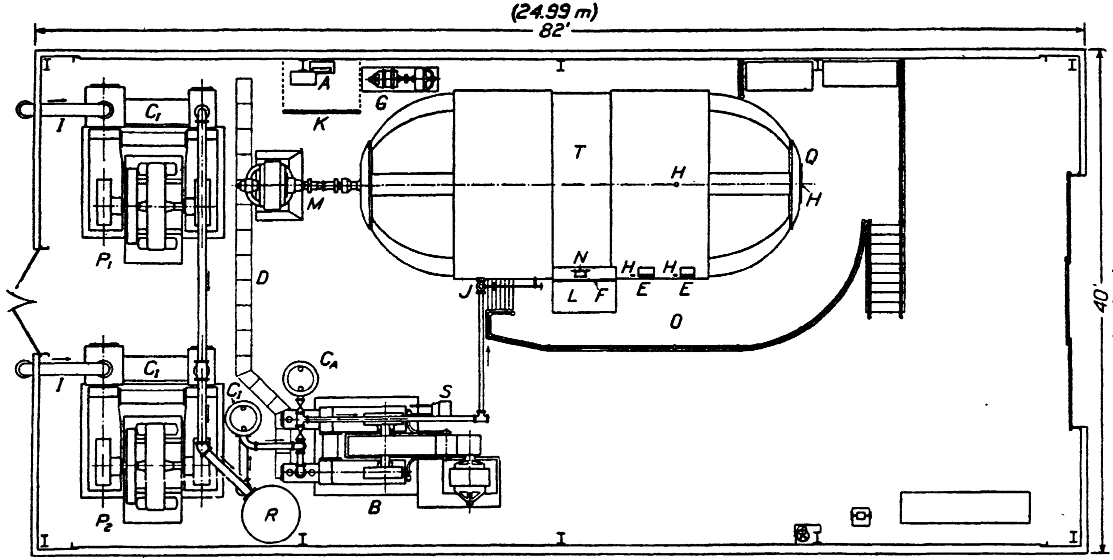


Fig. 1-13. Sectional elevation of variable density wind tunnel.

length of the chord may be used for this dimension). When the test is carried out under standard conditions as suggested (59° F and 29.92 in. mercury) the density of air (ρ) is 0.002378 slug/cu ft and the coefficient of viscosity (μ) is 0.000000373 slug/cu ft.

$$\text{Then} \qquad RN = \frac{\rho VL}{\mu} = \frac{0.002378 \times 100 \times 5}{0.000000373} = 3{,}150{,}000$$

A wind-tunnel model geometrically similar to the full-scale airfoil must be made. The dimensions of the model are: span 10 ft, and chord 0.5 ft. In order that the RN of the model in the test will be the same value desired above, it will be necessary to increase the velocity in the same ratio, *i.e.*, to have a velocity of 1000 ft/sec, or decrease the value of the kinematic viscosity to $^1/_{10}$ of that at sea level. It will be



A-Auto transformers
B-Booster compressor
C_I-Inter-cooler
C_A-After-cooler
D-Drainage trench
E-Control board
F-Terminal board
G-Motor-generator set
H-Peep-hole
I-Intake
J-Blow-off pipe to roof
K-Switch board
L-Desk
M-Drive motor
N-Gage
O-Operating platform
P_1-Primary compressor No. 1
P_2-Primary compressor No. 2.
Q-Door
R-Receiver
S-Starting box
T-Tank enclosing wind tunnel

Fig. 1-14. Floor plan of variable density wind tunnel and equipment.

seen at once that a velocity of 1000 ft/sec is highly impracticable; therefore a reduction of the kinematic viscosity is inevitable.

Remember the kinematic viscosity, ν, equals μ (coefficient of viscosity) divided by ρ (mass density of air). Since the coefficient of viscosity (μ) is independent of pressure, to reduce ν to 1/10 its value at sea level it is only necessary to increase the density (ρ) ten times.

This is the reason for the use of the variable-density wind tunnel. The N.A.C.A. has installed at Langley Field, Virginia, a variable-density wind tunnel in order to cope with this situation. See Figs. 1-13, 1-14. In the N.A.C.A. variable-density wind tunnel, tests are often run at a RN in excess of 3,000,000 and pressures greater than 20 atmospheres. Under these conditions wind-tunnel tests show excellent agreement with actual flight conditions; without such tests it is possible to come fairly close to the actual flight conditions from the experimental data, by using approximate curves of coefficient variation plotted against the RN.

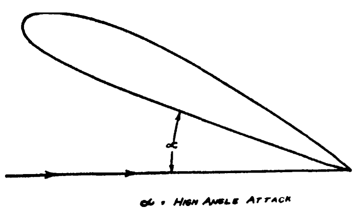


Fig. 1-15. Angle of attack.

It must be understood that the characteristics of the airfoil plotted against RN do not produce smooth curves. The only reliable prediction of full-scale performance can be obtained from a scale model in a variable-density tunnel where the density (ρ) can be increased to produce a RN that agrees with that of the full-scale wing.

PROBLEMS

1. Find the RN for an airplane wing of 5-ft chord, at 130 mph at standard conditions.
2. What is the RN for an airplane wing with 4 ft-6 in. chord moving 200 mph through standard air?
3. With air under standard conditions throughout, find the velocity at which a wind-tunnel test should be run on a model wing of 6-in. chord in order that the RN shall be the same as for a wing with a 6-ft chord at 100 mph.
4. Using a variable-density wind tunnel, under what pressure should tests be run on a model of 4-in. chord, air velocity 50 mph, in order that the RN shall be the same as for a full-size wing of 4-ft chord at 90 mph?

Relative Wind. Before anything further can be said about forces on an airfoil, it must be understood that *relative wind* is in the direction opposite to the motion of the airfoil. If the airfoil is moving horizontally forward, then the relative wind is horizontally backward. Likewise, if the airfoil is moving forward and upward, as the plane is climbing, the relative wind is downward and backward.

Angle of Attack is the angle the wing chord makes with the relative wind. (See Fig. 1-15.)

Forces on an Airfoil. The air striking an airfoil exerts a reactionary force on each little portion ΔS of the airfoil section. This pressure is considered *positive* if it is greater than atmospheric, and *negative* if less. (See Fig. 1-16, top.)

The air will flow smoothly over the upper surface at small angles of attack. This parallel motion of the molecules over each portion of the surface produces a force perpendicular to the surface. If the motion is not parallel, but toward the surface, the reaction will be a positive impact pressure sometimes referred to as "velocity head." (See Fig. 1-16, middle.)

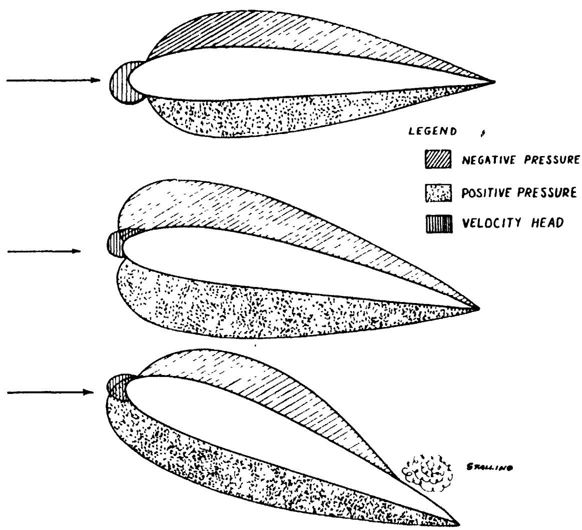


Fig. 1-16. Forces on an airfoil.

As the angle of attack, α, is increased the streamlines have a longer path to follow and therefore the velocity of the air over the top must increase. According to Bernoulli's theorem, an increase in velocity means a decrease in pressure, *i.e.*, more lift or suction will be produced with an increase in angle of attack. However, it must be understood that as the angle of attack is increased still more, the air is unable to follow the upper surface, as this would require too great a change in direction. When the streamlines can no longer flow smoothly over the surface, they begin to break away at the trailing edge and cause "burbling." This burbling will continue to become greater and will move forward if the angle of attack is increased more.

The burbling is actually the forming of small eddy currents which tend to stall the wing. Wherever burbling occurs, it indicates that the wing has lost most of its lifting action. (See Fig. 1-16, bottom.)

It is interesting to note how the small lifting forces on the airfoil may be added vectorially (see Fig. 1-17), and one resultant and its position can be shown.

The resultant force, at small angles of attack, is found to be back toward the trailing edge. As the angle of attack increases the resultant will move forward. The resolving of the lifting forces at each angle of attack will be different and the position of the resultant along the chord will be different. This position is called the *center of pressure* (C.P.) and will be discussed thoroughly later.

Characteristics of an Airfoil. The four quantities in which a designer is interested are:

Lift coefficient	Lift/Drag ratio
Drag coefficient	Center of pressure position

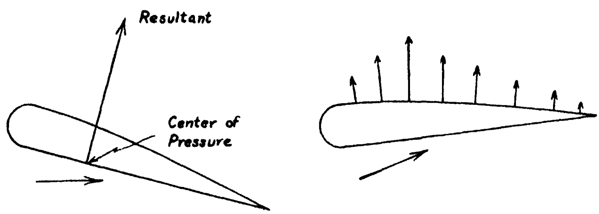
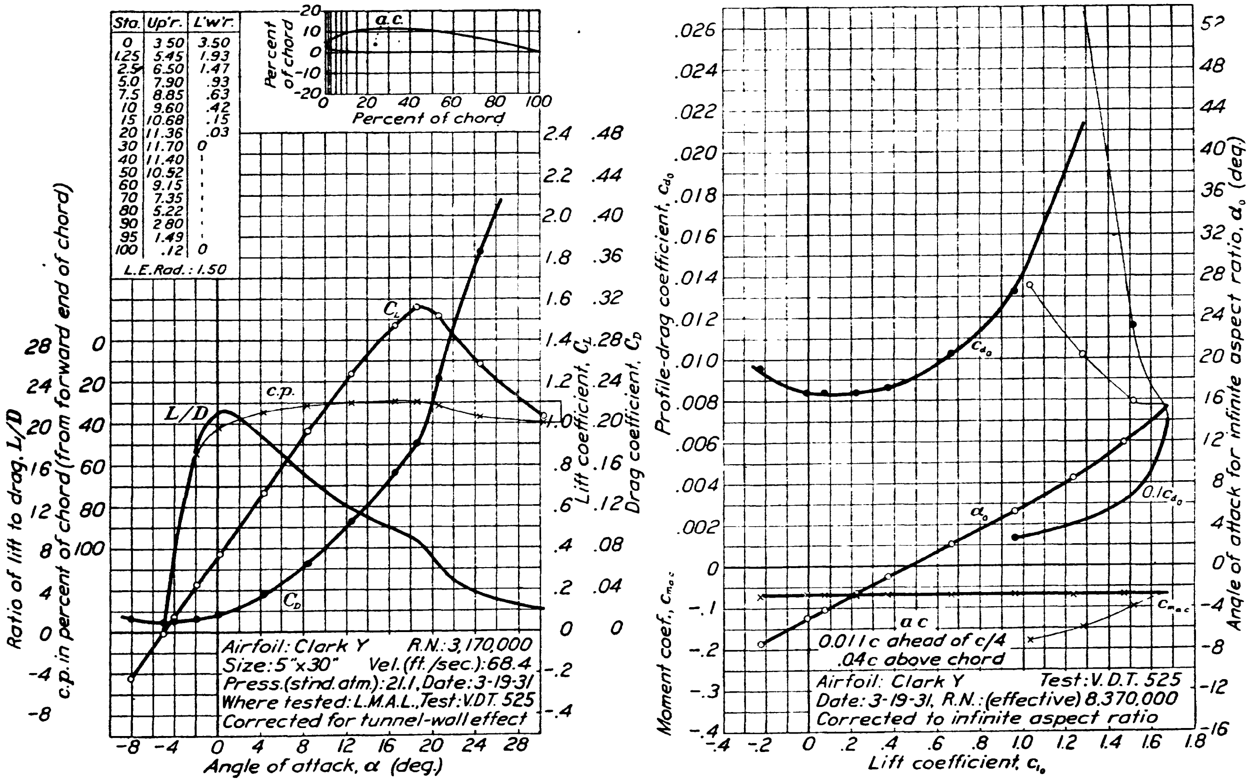


Fig. 1-17. Vector forces added to get resultant.

When the experimenters in the wind tunnel have tested the airfoil, they may plot these "characteristics" in several different ways. These plots are called the *characteristic curves* of an airfoil. The N.A.C.A. has prepared many interesting and informative data in the form of graphs and characteristic curves. (See Figs. 1-18 to 1-24.)

An examination of the characteristic curves for the various airfoils shows the lift coefficient, at most angles of attack, much larger than drag coefficients. The drag coefficients are plotted on a larger scale in order to be more easily read.

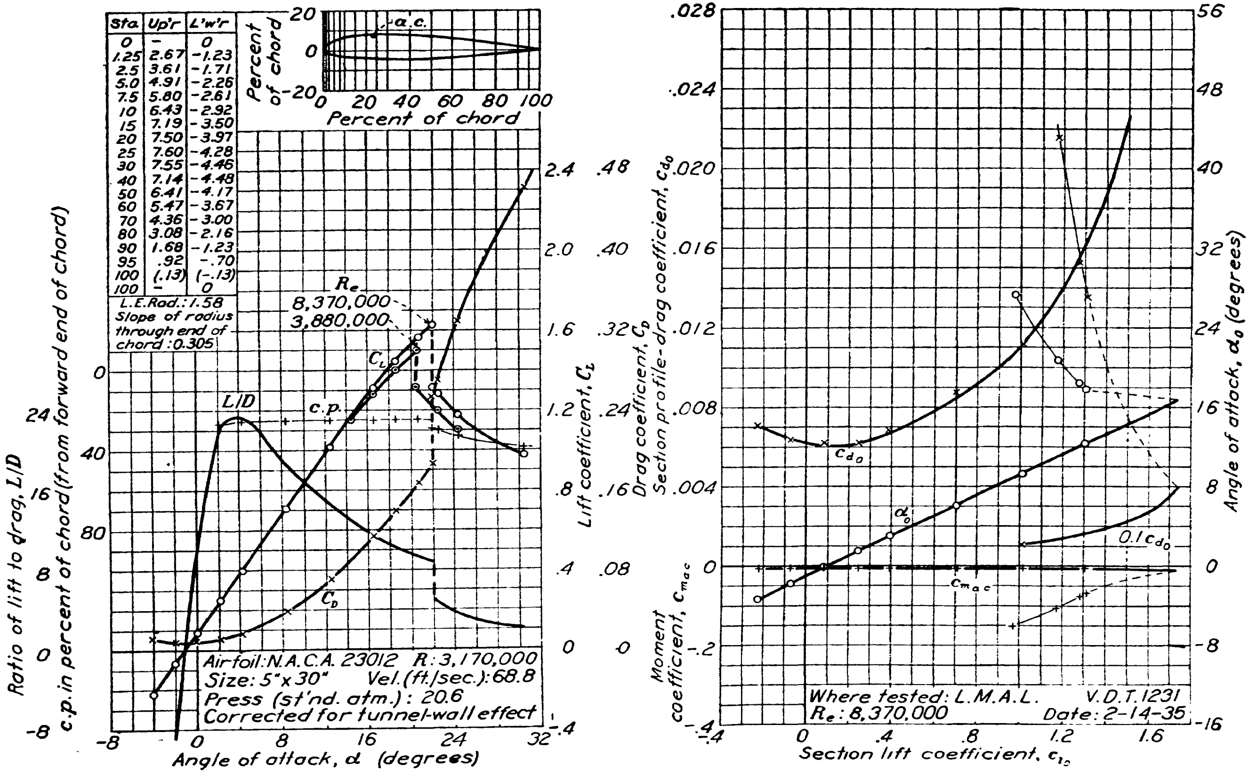
From the angle of zero lift, the curve of the coefficient of lift is usually a straight line, *i.e.*, the slope is constant. The lift coefficient curve begins to deviate from a straight line at larger angles of attack. There is one angle of attack at which the lift curve attains a maximum value. The angle at which this occurs is called the *critical angle* or the *burble point.* Usually the critical angle of attack is between 15° and 20°.



Sta.	Up'r.	L'w'r.
0	3.50	3.50
1.25	5.45	1.93
2.5	6.50	1.47
5.0	7.90	.93
7.5	8.85	.63
10	9.60	.42
15	10.68	.15
20	11.36	.03
30	11.70	0
40	11.40	
50	10.52	
60	9.15	
70	7.35	
80	5.22	
90	2.80	
95	1.49	
100	.12	0

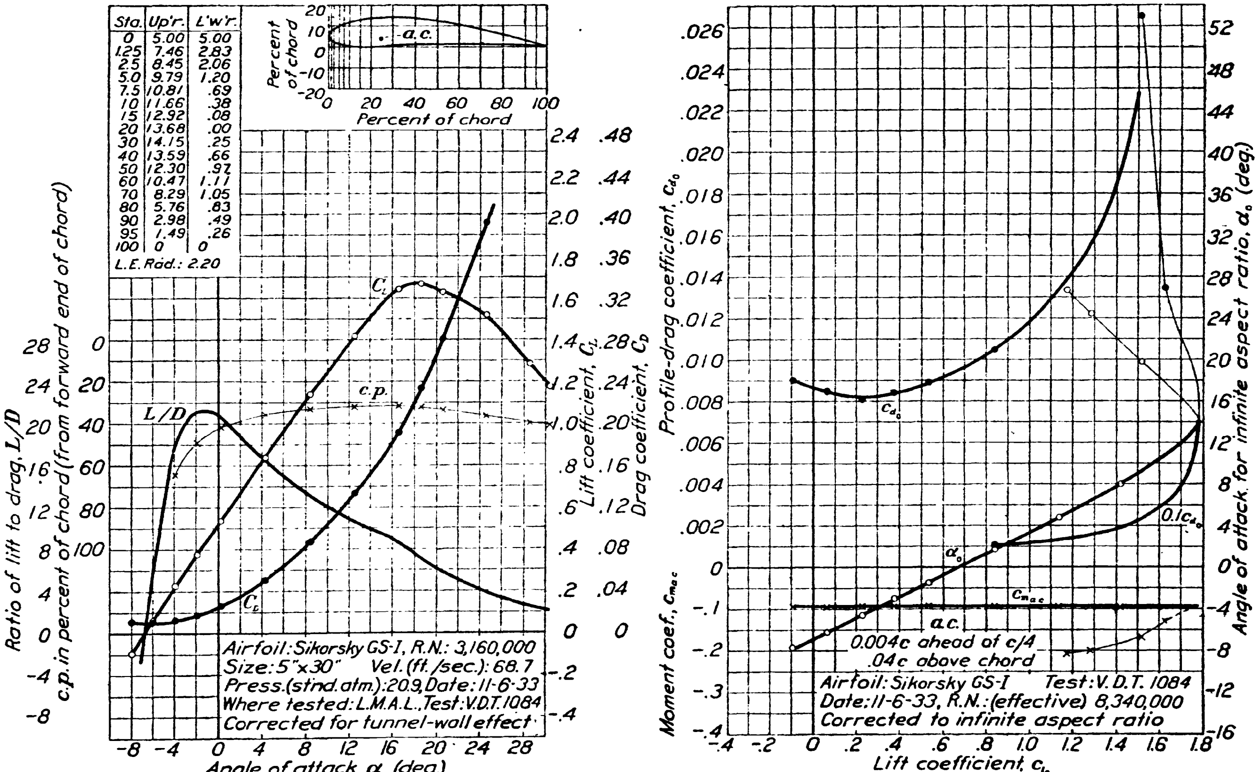
L.E. Rad.: 1.50

Fig. 1-18. Clark Y airfoil characteristics.



Sta.	Up'r	L'w'r
0	–	0
1.25	2.67	-1.23
2.5	3.61	-1.71
5.0	4.91	-2.26
7.5	5.80	-2.61
10	6.43	-2.92
15	7.19	-3.50
20	7.50	-3.97
25	7.60	-4.28
30	7.55	-4.46
40	7.14	-4.48
50	6.41	-4.17
60	5.47	-3.67
70	4.36	-3.00
80	3.08	-2.16
90	1.68	-1.23
95	.92	-.70
100	(.13)	(-.13)
100	–	0

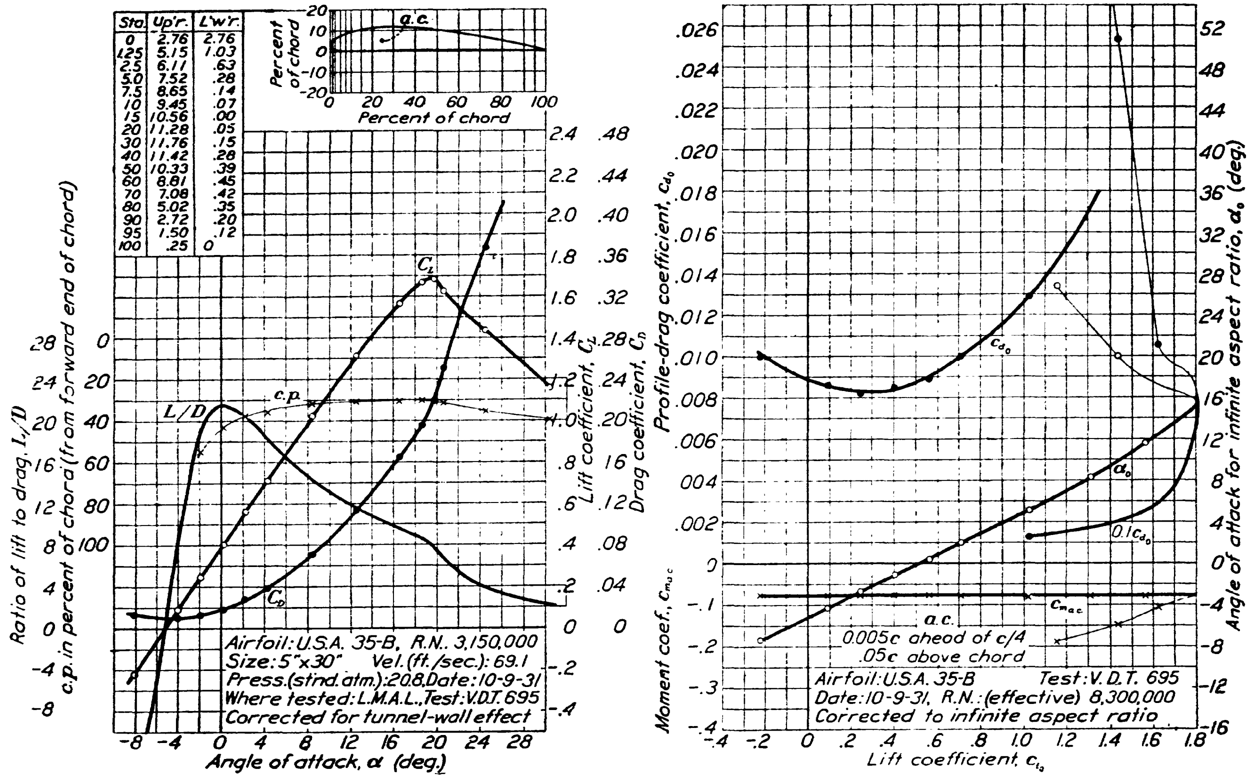
Fig. 1-19. N.A.C.A. 23012 airfoil.



Sta.	Up'r.	L'w'r.
0	5.00	5.00
1.25	7.46	2.83
2.5	8.45	2.06
5.0	9.79	1.20
7.5	10.81	.69
10	11.66	.38
15	12.92	.08
20	13.68	.00
30	14.15	.25
40	13.59	.66
50	12.30	.97
60	10.47	1.11
70	8.29	1.05
80	5.76	.83
90	2.98	.49
95	1.49	.26
100	0	0

L.E. Rad.: 2.20

Fig. 1-20. Sikorsky GS-I airfoil, N.A.C.A.



Sta.	Up'r.	L'w'r.
0	2.76	2.76
1.25	5.15	1.03
2.5	6.11	.63
5.0	7.52	.28
7.5	8.65	.14
10	9.45	.07
15	10.56	.00
20	11.28	.05
30	11.76	.15
40	11.42	.28
50	10.33	.39
60	8.81	.45
70	7.08	.42
80	5.02	.35
90	2.72	.20
95	1.50	.12
100	.25	0

Fig. 1-21. U.S.A. 35-B airfoil, N.A.C.A.

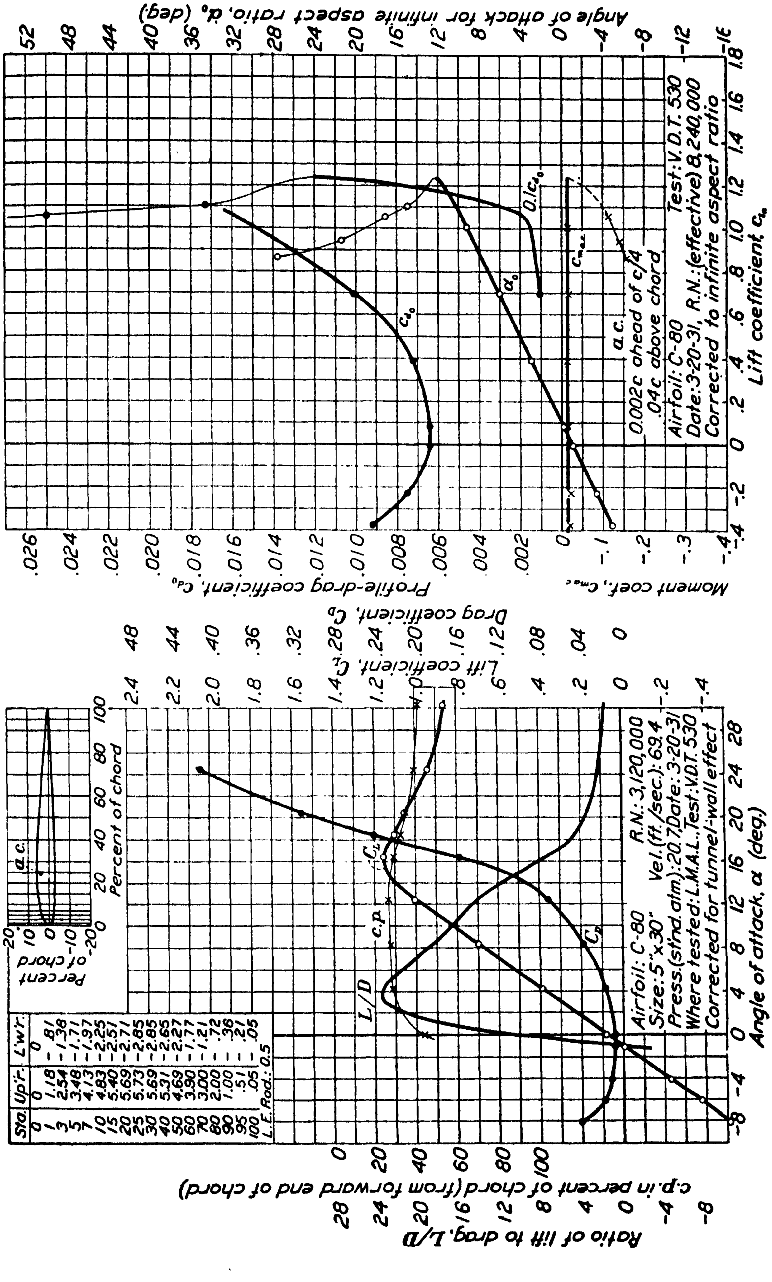
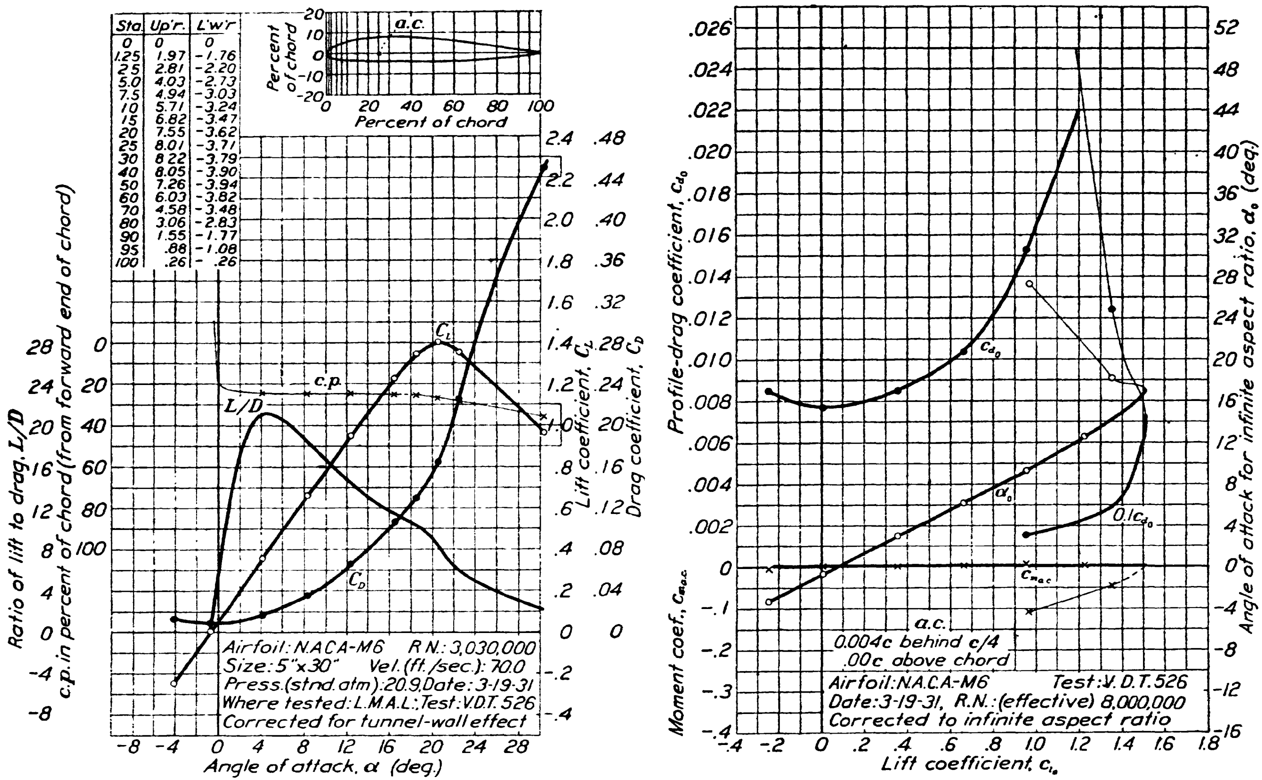


Fig. 1-22. C-80 airfoil, N.A.C.A.



Sta.	Up'r.	L'w'r
0	0	0
1.25	1.97	-1.76
2.5	2.81	-2.20
5.0	4.03	-2.73
7.5	4.94	-3.03
10	5.71	-3.24
15	6.82	-3.47
20	7.55	-3.62
25	8.01	-3.71
30	8.22	-3.79
40	8.05	-3.90
50	7.26	-3.94
60	6.03	-3.82
70	4.58	-3.48
80	3.06	-2.83
90	1.55	-1.77
95	.88	-1.08
100	.26	-.26

Fig. 1-23. N.A.C.A.-M6 airfoil.

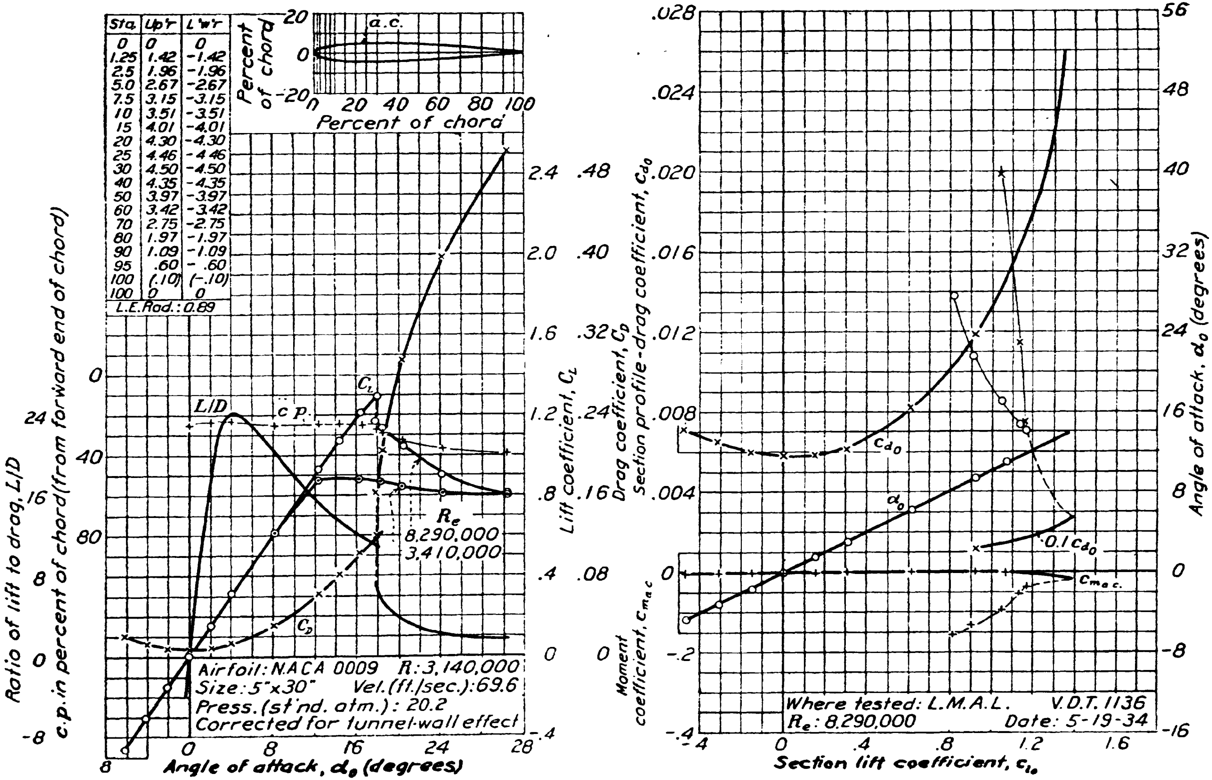


Fig. 1-24. N.A.C.A. 0009 airfoil.

Angle of Attack versus C_L and C_D, etc. The usual way of representing the data for a specific test is the plots of C_L, C_D, C_L/C_D and C.P. against the angle of attack. See Fig. 1-18.

Lift Formulas with Absolute Coefficients. There is a useful reaction generated by an airfoil moving through the relative wind, namely, *lift.* The formulas for lift with absolute coefficients, as used by N.A.C.A., are as follows:

Lift = Coefficient (C_L) $\times$ $^1/_2$ mass density of air $\times$ area $\times$ (velocity)^2.
Lift is in lb or kg.
Density of air in slugs per cu ft or metric slugs per cu meter.
Area is in sq ft, or sq m.
Velocities are in ft/sec or m/sec.

The expression for lift is usually found thus:

$$\text{Lift } (L) = C_L\, \rho/2\, SV^2 = C_L qS$$

where $q = \rho/2\ V^2$ called *dynamic pressure*, expressed in lb/sq ft.

The expression *dynamic pressure* corresponds to the "velocity head" in hydraulics and is the pressure indicated by a pitot-static tube placed at the corresponding point in the airstream. This pressure is the pressure which would be indicated at the upstream side of a flat plate normal to the wind. It should be pointed out that the coefficients of C_L, C_D, C_M, are independent of the system of units whether English or metric, so long as consistent units are employed.

Drag Formula with Absolute Coefficients. Designers must be interested not only in the *lift* of the airfoil but also the price they must pay for it: *drag.* Drag is the resistance encountered by a solid moving through the air. It is extremely important in the calculation of the performance of a newly designed airplane and is always included with all of the aforementioned characteristic curves.

In reality there are three different kinds of drag (*profile, induced,* and *parasite* drag). When the total drag of an airplane is spoken of, all of these are referred to.

The formula for wing drag is:

$$D = C_D\, \rho/2\, SV^2$$

where D = drag in lb or kg
ρ = mass density of air in slugs per cu ft or metric slugs per cu meter
S = area of wing in sq ft or sq m
V = velocity in ft/sec or m/sec

In the Introduction an illustration of an airplane, in unaccelerated level flight, has the lift vector equal to the gravity vector, *i.e.*, in

horizontal flight the weight of the plane equals the lift. If the lift is greater than the weight, the plane will rise; likewise, if the weight is greater the plane will lose altitude. Thus for horizontal flight:

$$W = C_L \, \rho/2 \, SV^2$$

where W = weight of plane in lb
S = area in sq ft (wing)
V = velocity in ft/sec

Hence, with a given airfoil at standard conditions, the lift on an airplane is dependent on two variables: the angle of attack and the velocity. With an increase of angle of attack, up to the stalling point, the lift will increase; and the *lift* will increase as the square of the velocity.

Example: What is the required weight for an airplane to fly level with a Clark Y wing 250 sq ft in area, at 6° angle of attack and air-speed of 100 mph at standard sea level conditions?

Solution: From Fig. 1-18,

$$C_L \text{ at } 3° \text{ angle of attack} = 0.79$$
$$100 \text{ mph} = 146.7 \text{ ft/sec}$$

$$W = 0.79 \times \frac{0.002378}{2} \times 250 \times (146.7)^2 = 5050 \text{ lb.}$$

Wing Loading is another important factor in airplane performance. It is the ratio of the total weight of the airplane to the area of the wing (W/S), and is expressed in pounds per sq ft. Then

$$V = \sqrt{\frac{W}{S}} \sqrt{\frac{1}{C_L \, \rho/2}}$$

It can be seen now that, for a given angle of attack, the cruising speed depends on the square root of the wing loading.

Example: If an airplane has a wing loading of 16 lb/sq ft, and a Clark Y wing at 4° angle of attack, find the required airspeed for level flight.

Solution: From Fig. 1-18 at 4° angle of attack, $C_L = 0.649$

$$\begin{aligned} V &= \sqrt{16} \sqrt{\frac{1}{0.649 \times 0.001189}} \\ &= 144.2 \text{ ft/sec} \\ &= 98.3 \text{ mph} \end{aligned}$$

Other variations of the lift equation may be used. For example, for level flight it is easy to find the proper angle of attack for a certain given cruising speed and wing loading.

Example: A plane with a wing loading of 9 lb/sq ft and cruising

speed of 120 mph must fly at what angle of attack if designed with a Clark Y wing?

Solution:

$$C_L = \frac{W}{S} \times \frac{1}{\rho/2\ V^2}$$

$$= 9 \times \frac{1}{0.001189 \times \left(120 \times \frac{44}{30}\right)^2}$$

$$= 0.244$$

Then in Fig. 1-18 is found for C_L of 0.244 an angle of attack of $-1.7°$.

PROBLEMS (Based on Standard Conditions)

1. An airplane weighs 2000 lb and has a wing loading of 20 lb/sq ft. Find the airspeed for level flight if the airfoil is operating at a value of $C_L = 0.42$.
2. What is the wing loading of an airplane if the velocity is 100 ft/sec, and $C_L = 0.80$?
3. An airplane has a wing loading of 15 lb/sq ft and is cruising at 98 mph. What is the angle of attack of the Clark Y wing?
4. Find the wing loading of an airplane flying level at 120 ft/sec and the $C_D = 0.04$ for Clark Y.
5. It is desired that a fighter plane be able to fly at 60 mph in level flight. What should be its wing loading if a Clark Y wing is used at 10° angle of attack?
6. An airplane has a Clark Y wing 420 sq ft in area, and a gross weight of 7000 lb. If it cruises at 140 ft/sec, find the angle of attack.
7. Two airplanes are flying at the same value of lift coefficient ($C_L = 1.10$). Airplane A has a wing loading of 16 lb/sq ft and airplane B has a wing loading of 9 lb/sq ft. Which is the faster moving airplane?

Landing Speed. It is evident from the formula

$$V = \sqrt{\frac{W}{S}}\sqrt{\frac{1}{C_L\ \rho/2}}$$

that with a given wing loading (fixed weight and fixed wing area), the lift coefficient varies inversely as the square of the velocity. At small angles of attack the velocity is great and at large angles of attack the velocity is less. So, the slowest velocity occurs when the lift coefficient is maximum. It follows that, when the airfoil is at the angle of attack for maximum lift coefficient, the stalling speed is reached. To land an airplane correctly, a pilot must stall the plane just before the wheels touch the ground. With this in mind, it is an easy matter to find the landing speed of any airplane.

Example: Find the landing speed of a plane with a Clark Y wing of 250 sq ft and gross weight of 2000 lb.

Solution: From Fig. 1-18 the height of the lift curve has a value of $C_{L\,max} = 1.56$:

$$V_{min} = \sqrt{\frac{W}{S} \times \frac{1}{C_{L\,max} \times \rho/2}}$$

$$= \sqrt{\frac{2000}{250} \times \frac{1}{1.56 \times 0.001189}}$$

$$= 65.9 \text{ ft/sec}$$

$$= 44.9 \text{ mph}$$

Sometimes in heavier airplanes, *i.e.*, with greater wing loading, high lift devices are used which increase the maximum lift coefficient and allow lower landing speeds than could be obtained from a plain wing.

Horsepower Required by Wing. Drag is the amount of force, in pounds, with which the wing resists forward motion through the air. Drag acts backward and is balanced by the forward thrust of the propeller. If the airplane is flying level at a constant airspeed, the horizontal forces of thrust and drag equal zero, *i.e.*, they are in equilibrium. If the throttle is eased forward, the airspeed will increase and there will be an acceleration. Drag increases as the square of the velocity. Therefore, as the speed is increased the new velocity is reached and the total drag equals the thrust. There is still no unbalanced force and the airplane will fly at that new speed.

If the throttle is closed slightly, there will be momentarily a deceleration because of the greater backward drag. At a slower forward speed there will be a reduction of drag. The drag will again be equal to the thrust and equilibrium of the horizontal forces will result. If drag is expressed in pounds and the velocity in ft/sec, the product of the two will give the power required for the wing in ft-lb/sec. In other words, for a given airfoil at a fixed airspeed, it is possible to find the power required to move the wing through the air. By definition, one horsepower (HP) is equal to 33,000 ft-lb/min or 550 ft-lb/sec. Therefore the drag in pounds multiplied by the velocity in ft/sec and divided by 550 will give the HP required for the wing.

Thus

$$HP_{req} = \frac{\text{drag} \times \text{velocity}}{550}$$

where drag in lb; V in ft/sec

and Drag $= C_D\, \rho/2\, SV^2$

then $$HP_{req} = \frac{C_D\, \rho/2\, SV^3}{550}$$

It follows that the horsepower required to pull the wing through the air varies as the cube of the velocity. Economy of operation and per-

formance is of utmost importance to the commercial aircraft designer: An airplane should be as *clean aerodynamically as possible, i.e.*, to cut down on the drag.

Example: Find the drag of a Clark Y wing with a wing area of 300 sq ft and an airspeed of 90 ft/sec at 8° angle of attack. What is the HP required?

Solution: From Fig. 1-18 at 8° angle of attack $C_D = 0.06$

$$\text{Drag} = 0.06 \times \frac{0.002378}{2} \times 300 \times (90)^2$$

$$= 173.4 \text{ lb}$$

then

$$HP_{req} = \frac{173.4 \times 90}{550}$$

$$= 28.3\ HP$$

It is important to point out that the total HP required for the complete airplane will be considerably greater than for the wing alone because of the resistance of the fuselage, landing gear, etc., which will be studied in a later chapter.

PROBLEMS

1. Find the HP required by a Clark Y wing if the airspeed is 100 mph, wing area 250 sq ft and the wing is operating at 4° angle of attack?
2. What will be the drag of a 300 sq ft Clark Y wing if the angle of attack is −1° and the airspeed is 220 mph? What HP is required for the wing?
3. Find the least drag of a Clark Y wing if the airplane weighs 4000 lb. If the airplane is flying at 300 mph what will be the HP required?
4. An airplane is operating at an angle of attack of $C_D = 0.09$ and the wing area is 225 sq ft. If the airplane flies 175 mph what will be the drag? What will be the HP required for level flight?
5. What is the HP required to move a Clark Y wing 350 sq ft in area at 10° angle of attack and an airspeed of 215 ft/sec?

Altitude Flying. Thus far, lift and drag and horsepower required have been determined at sea-level conditions. At altitudes the density of the air is less than at sea level. Lift and drag, respectively, are equal to a coefficient times the air density times the wing area and the square of the velocity. It is clear, then, with a decrease of air density there will be a decrease of lift and drag. But regardless of altitude, if an airplane is to fly level, the lift must equal the weight. With a fixed wing area, when an airplane is flown at altitudes, the lift coefficient or velocity must be increased to make up for the decrease in density.

Suppose that the angle of attack is fixed, *i.e.*, the value of C_L and C_D remain unchanged in the equation

$$L = C_L \, \rho_0/2 \, S V_0^2$$

where ρ_0 = density at sea level
V_0 = velocity at sea level

and
$$L = C_L \, \rho/2 \, S V^2$$

where ρ = density at altitude
V = velocity at altitude

For level flight, lift equals weight

thus
$$W = C_L \, \rho_0/2 \, S V_0^2$$

and
$$W = C_L \, \rho/2 \, S V^2$$

then
$$V^2/V_0^2 = \rho_0/\rho$$

and
$$V^2 = \rho_0/\rho \; V_0^2$$

therefore
$$V = V_0\sqrt{\rho_0/\rho}$$

It can be seen from the above expression that the velocity at altitude is greater than at sea level.

$$\text{Drag at sea level} = D_0 = C_D \, \rho_0/2 \, S V_0^2$$
$$\text{Drag at altitude} = D = C_D \, \rho/2 \, S V^2$$

but
$$V^2 = \rho_0/\rho \; V_0^2$$

therefore
$$D = C_D \, \rho/2 \, S \, \rho_0/\rho \; V_0^2$$
$$= C_D \, \rho_0/2 \, S V_0^2$$

then
$$D_0 = D$$

Regardless of the altitude, if the angle of attack remains the same, the drag of the airfoil remains constant. This is because at altitudes the molecules of the air are farther apart and this necessitates a greater airspeed. The increase of airspeed takes care of the decrease in density.

The horsepower required for a wing is the product of drag and velocity divided by 550 ft-lb. Then let HP_0 equal the horsepower at sea level and HP equal horsepower at some altitude at the same angle of attack.

$$HP_0 = \frac{D_0 \times V_0}{550}$$

$$HP = \frac{D \times V}{550}$$

and
$$D = D_0, \; V = V_0\sqrt{\rho_0/\rho}$$

then $$HP = \frac{D_0 \times V_0}{550} \sqrt{\rho_0/\rho}$$

therefore $$HP = HP_0 \quad \sqrt{\rho_0/\rho}$$

Since ρ_0 is density at sea level and ρ is density at altitude it can be seen that the expression $\sqrt{\rho_0/\rho}$ is greater than unity and, therefore, the horsepower required at altitude will be greater than that at sea level.

Example: An airplane with a Clark Y wing with area of 300 sq ft is flying at sea level at 120 ft/sec. If the airplane weighs 5000 lb, what are the wing drag and HP required for the wing? If the airplane is flying at 12,000 ft at 120 ft/sec what are the wing drag and HP required?

Solution: For sea-level conditions:

$$C_L = \frac{5000}{0.001189 \times 300 \times (120)^2}$$

From Fig. 1-18 if $C_L = 0.973$

therefore $$\alpha = 8.7°$$

and $$C_D = 0.068$$

$$\begin{aligned} \text{Drag} &= C_D\, \rho/2\, SV^2 \\ &= 0.068 \times \frac{0.002378}{2} \times 300 \times (120)^2 \\ &= 349 \text{ lb} \\ HP_0 &= \frac{349 \times 120}{550} \\ &= 76.25\ HP \end{aligned}$$

and at 12,000 ft

$$\begin{aligned} C_L &= \frac{5000}{\frac{0.001648}{2} \times 300 \times (120)^2} \\ &= 1.41 \end{aligned}$$

therefore $$\alpha = 15.4°$$

and $$C_D = 0.136$$

$$\begin{aligned} D &= 0.136 \times \frac{0.001648}{2} \times 300 \times (120)^2 \\ &= 484 \text{ lb} \\ HP &= \frac{484 \times 120}{550} \\ &= 105.6\ HP \end{aligned}$$

It can be seen that the HP required will be greater at 12,000 ft.

If the angle of attack and velocity are unchanged, it is possible that the decrease in density would be great enough for the drag and HP required to be less at altitude.

PROBLEMS

1. An airplane has a top speed at sea level of 150 mph. What will be the top speed at 5000 ft? At 10,000 ft?

2. An airplane at 20,000 ft has an indicated airspeed of 300 mph. What is the true airspeed?

3. An airplane with a C-80 airfoil of 265 sq ft is flying at sea level at 230 ft/sec. If the airplane weighs 4000 lb, what are the wing drag and HP required for the wing? If the airplane is flying at 20,000 ft at 230 ft/sec, what are the wing drag and HP required?

4. An airplane has a Clark Y wing 375 sq ft in area. It flies 120 ft/sec at 3° angle of attack. What are the lift, wing drag, and HP required at sea level? What are the lift, wing drag, and HP required at 10,000 ft altitude?

5. An airplane whose wing loading is 15 lb per sq ft has a Clark Y wing and is flying 200 mph at sea level. (a) What is the angle of attack? (b) What is the angle of attack if flying at 12,000 ft?

6. An airplane weighs 5750 lb, has a Clark Y wing, and flies at a 3° angle of attack and a velocity of 147 ft/sec. What are the wing area and HP required: (a) at sea level; (b) at 15,000 ft?

Lift-Drag Ratio. It is evident that, for every angle of attack and at a certain airspeed, there must be a reactionary resultant force. It has been customary to resolve this resultant into its two components, namely, lift and drag. As has been seen in the various *characteristic curves* of different airfoils, the lift coefficient is always greater than the corresponding drag. Thus a closer inspection of these characteristic curves shows the lift curve as practically a straight line up to the point of stalling, *i.e.*, up to the maximum lift ($C_{L\ max}$) the slope is constant. The drag curve acts somewhat differently, and at small angles of attack the curve does not immediately ascend but remains practically horizontal and then increases rapidly. (See Fig. 1-25.) In other words, at the angle of attack of minimum drag coefficient, it will be found that an increase of a few degrees of angle of attack will cause only a slight increase in drag coefficient and a considerable increase in the lift coefficient. This is an important fact in airfoil selection since it is every designer's wish to get the most lift for the least drag. All airfoils, regardless of their relative merits, have, at some specific angle of attack, the most lift for the least drag, namely, the maximum lift-drag ratio, or, as it is sometimes called, the "best L over D." Actually the lift-drag ratio (L/D ratio) is the tangent of the angle that the resultant force makes with the relative wind (Fig. 1-26). As the angle of attack is increased beyond zero lift

(where L/D is zero) the L/D increases rapidly, reaching its maximum value at the angle of attack from approximately 0° to +4°, depending, of course, on the particular airfoil.

As the angle of attack increases beyond the maximum L/D, the L/D decreases slowly. It becomes zero again when the angle of

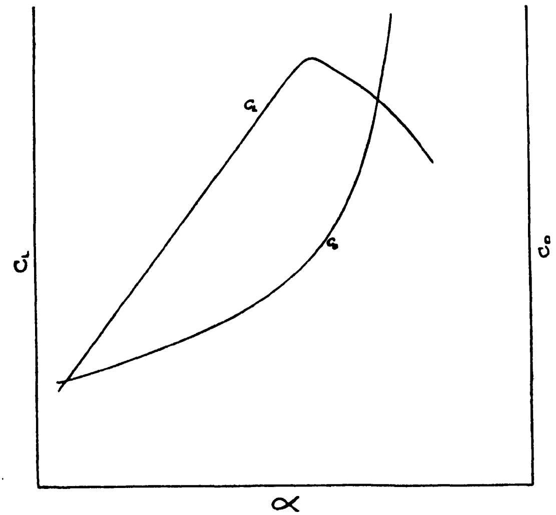


Fig. 1-25. Typical lift and drag curves.

attack approaches 90°, since, under this condition, it becomes a flat plate and the only force present is the drag.

The L/D curve is plotted for every airfoil and is considered as one of the most important characteristics.

Polar Curves. In Fig. 1-26 it has been pointed out that the best L/D may be found by largest value of the tangent of α that can be produced in the flight range. Undoubtedly, it follows that the polar diagram can be of great value in determining the best L/D. Instead of the customary plots of C_L and C_D against angle of attack, the

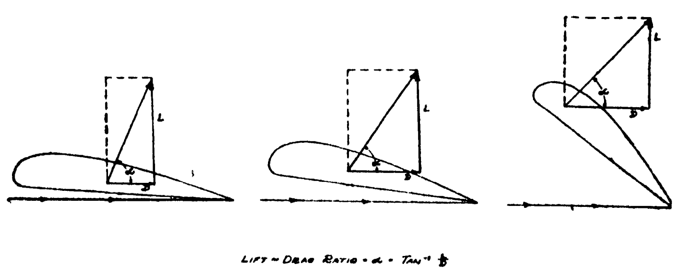
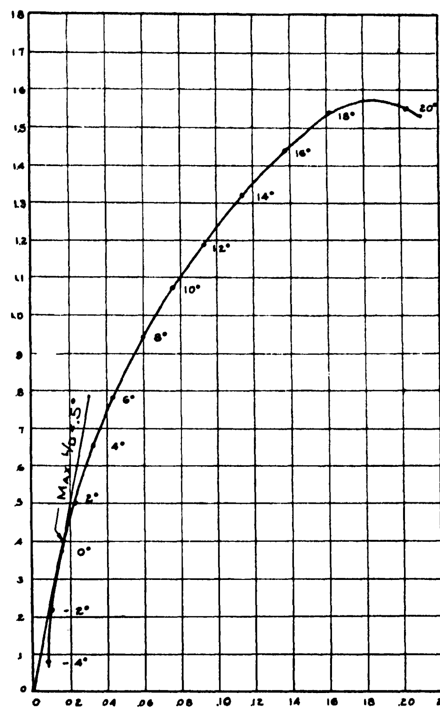


Fig. 1-26. Diagram of best lift-drag ratio.

polar curve is used with the lift coefficient plotted against the drag coefficient (Fig. 1-27). The curve itself is the angle of attack. The lift coefficients are the ordinates and the drag coefficients the abscissas. When the lift and drag coefficients are plotted, the slope of the radius vector drawn from the origin to any point on the curve is the L/D ratio for that particular point. The length of the radius vector is the magnitude of the coefficient of the resultant force on the wing; in other words, to find the force on the wing it will only be



Polar Curve for Clark Y Airfoil

Fig. 1-27. Polar curve for Clark Y airfoil.

necessary to multiply this coefficient by $\rho/2\ SV^2$. Since the lift coefficients are usually several times larger than the drag coefficients, it is best to use different scales. In this case the radii vectors have no significance until they have been multiplied by the ratio of the scales.

To determine the best L/D of the airfoil, it is only necessary to extend a straight line from the origin of the coordinates tangent to the curve. At the point of tangency, the corresponding angle of attack for maximum L/D will be found. That is, the line drawn from the origin of the coordinates making the largest angle with the abscissa

will give the highest C_L/C_D ratio. With the aid of the polar curve of the Clark Y airfoil, Fig. 1-27, the maximum L/D is noted to be at 0.5° angle of attack. By use of the polar curve, the designer may

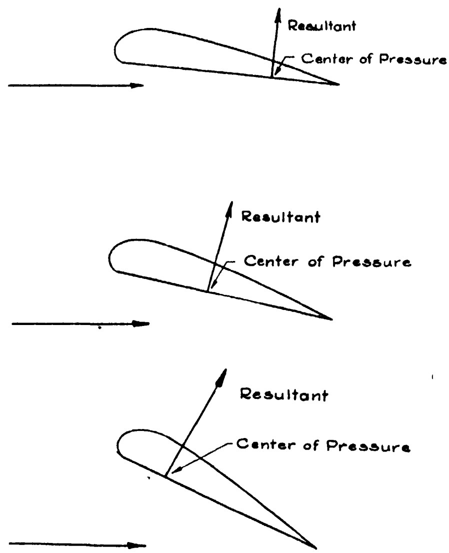


Fig. 1-28. Center of pressure travel with angle of attack.

quickly find at a glance, a value that would otherwise necessitate several other separate curves.

PROBLEMS

1. Plot the polar curve for the C-80 airfoil and find the best L/D by tangent line.
2. Plot the polar curve for the Sikorsky GS-1 airfoil and draw tangent line to locate best L/D.
3. Plot the polar curve for the U.S.A. 35-B airfoil and find the best L/D by tangent line.
4. Plot the polar curve for the N.A.C.A. M-6 and find by tangent line best L/D.
5. Plot the polar curve for the N.A.C.A. 23012 and find best L/D by tangent line.

Center of Pressure. The position on the chord, through which the resultant of the forces acts, is called the *center of pressure* (abbreviated C.P.). The location of the C.P. is given as one of the characteristic curves and is given in percentage of the chord from the leading edge. For all airfoils having greater upper camber than lower, the C.P. location is practically the same. For angles of attack near that for zero lift, the C.P. is near the trailing edge, and in some extreme cases

may be found to be entirely off the trailing edge. As the angle of attack is increased, the C.P. moves forward (Fig. 1-28). The most forward position of C.P. occurs very near the stalling angle ($C_{L\ max}$). The most forward position of C.P. for cambered airfoils occurs at about 30 percent of the chord from the leading edge. The C.P. curve is usually plotted on the same graph as the lift and drag coefficient.

When the C.P. moves forward with an increase in angles of attack, the *center-of-pressure travel* is considered *unstable*. In other words, if by some means the angle of attack was suddenly increased from a former balanced condition, the C.P. would move forward and, instead of the airfoil tending to go back to its original position, this would tend to lift the leading edge still farther. This would continue until stalling of the wing would occur. Likewise, if the angle of attack was depressed from a previously balanced condition, it would

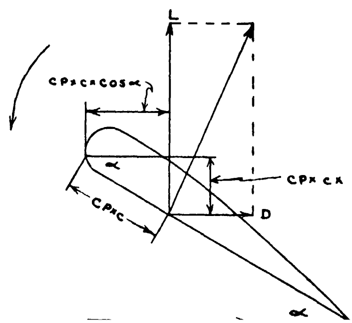


Fig. 1-29. Moment about leading edge.

cause the C.P. to move backward and hence the airfoil would tend to dive even more. It is interesting to note that even though cambered airfoils are considered, by themselves, as unstable, they have been used successfully on stable airplanes.

With flat plates, the C.P. travel is just the reverse of the cambered airfoil and it is considered *stable*. For symmetrical airfoils such as the N.A.C.A. 0009, having both surfaces convex, there is practically no C.P. travel. The airfoils designated as *M* sections, after their designer, Dr. Max Munk, have been developed which have very little or no C.P. travel. It will be found, in the chapter on Stability, that it is most desirable, for balance considerations, to have only a small C.P. travel but it must also be kept in mind that it may be very undesirable to have no C.P. travel whatever. The reason is, in a wing of two-spar construction, there may be too much stress on one spar and practically none on the other. The criterion is to have enough C.P.

travel so that the load may shift to one spar and then the other in different flight attitudes, *i.e.*, to distribute the load more evenly.

Moments about Leading Edge. One of the more critical and difficult factors in aircraft design is stress analysis. One particular difficulty arises in the inability of the engineer to predict accurately the exact strength requirements for each part. Not only is it difficult to predict the most critical loads and stresses in actual flight, but it was at one time difficult to calculate them.

The problem of *longitudinal stability* and balance are of great importance. If proper tail designs, wing taper, etc., are to be incorporated into the design, some means of determining the position of forces and their magnitude must be used.

The principal load-bearing members in the wings are the spars. The spars must be designed to take both direct and lateral loads, thus serving in the capacity of columns as well as beams. To find the moment (force times distance) about the leading edge, the sum of the moments of the two components, lift and drag, may be used (Fig. 1-29):

$$\text{Moment about the leading edge} = M_{LE} = (C.P. \times c \times \cos \alpha \times L) + (C.P. \times c \times \sin \alpha \times D)$$

where M_{LE} = moment in ft-lb
$C.P.$ = percentage of chord length
c = length of chord in ft
L and D are in lb.

In a more familiar expression the moment about the leading edge may be written:

$$M_{LE} = C_{MLE} c \, \rho/2 \, SV^2$$

where $C_{MLE} = (C.P.)(C_L \cos \alpha + C_D \sin \alpha)$

The angle of attack is never very great so that, for fairly close approximation, the expression may be thus:

$$C_{MLE} = -(C.P.)(C_L)$$

The minus sign is used to indicate diving moments because nosing-up moments are considered positive by N.A.C.A. The expression may now be written thus:

$$M_{LE} = -(C.P.) \times C_L \times c \times \rho/2 \, SV^2$$

Example: Find the moment about the leading edge of a rectangular Clark Y airfoil of 7-ft chord and 42-ft span at a 6° angle of attack with an airspeed of 110 ft/sec.

Solution: From Fig. 1-18 when angle of attack is 6°

$$C_L = 0.79, \; C.P. = 33\%$$

Then moment about the leading edge is

$$M_{LE} = -0.33 \times 0.79 \times 7 \times \frac{0.002378}{2} \times 294 \times (110)^2$$

$$= -7730 \text{ ft-lb}$$

Example: At an 8° angle of attack an airfoil has a $C_L = 1.1$, $C_{M_{LE}} = -0.32$. Find the C.P. by the approximate method.

Solution:

$$C_{M_{LE}} = -(C.P.) \times (C_L)$$

$$C.P. = \frac{0.32}{1.1}$$

$$= 0.29\% \text{ of chord from leading edge}$$

PROBLEMS

1. Find the moment about the leading edge of a Clark Y airfoil with 8-ft chord and 48-ft span, −3° angle of attack, and airspeed of 140 mph.
2. By approximate method, find the moment coefficient about the leading edge of a Clark Y airfoil at 4° angle of attack. If the chord of the rectangular wing is 9 ft and the span is 54 ft, find the moment about the leading edge when the velocity is 200 mph.
3. Plot moment coefficient versus angle of attack for Clark Y airfoil.
4. What vertical force is necessary to be applied 4 ft back from the leading edge to prevent rotation of a Clark Y wing of 6-ft chord and 36-ft span if the angle of attack is 1° and the airspeed is 80 mph?
5. Find the moment about the leading edge of a wing whose area is 400 sq ft and span is 60 ft. The airfoil is an N.A.C.A. 23012 at 5° angle of attack and the velocity of the airplane is 173 mph.
6. Plot moment coefficient versus angle of attack for N.A.C.A. 23012.
7. At a 7° angle of attack an airfoil has a $C_L = 1.3$ and moment coefficient about the leading edge equals −0.46. Find the C.P. by the approximate method.
8. At 16° angle of attack an airfoil has $C_L = 1.3$ and moment coefficient about the leading edge is equal to −0.325. By the approximate method find the C.P.
9. Find the moment about the leading edge of a U.S.A. 35-B airfoil if the chord is 9 ft and span 64 ft, $C_L = 0.39$, $C.P. = 0.44$ at 0° angle of attack and airspeed of 120 mph.
10. Find the moment about the leading edge of a Clark Y airfoil 30-ft span, 5-ft chord, −1° angle of attack, and airspeed of 75 mph.

Aerodynamic Center. In practically all characteristic curves, reference is made to the moment about the quarter-chord, sometimes called the moment about the *aerodynamic center* (a.c.). It has been proved in actual flight tests and in wind-tunnel tests that when the wing is flown at the angle of attack for zero lift ($C_L = 0$) the twisting moment

becomes extremely high. The only time the wing can fly at zero lift, for any appreciable time, is in a vertical dive. When C_L equals zero, the moment coefficient about any point is equal to the moment coefficient about the quarter point. It follows that, at the angle of attack for zero lift, the moment coefficient about the leading edge is equal to the moment coefficient about the quarter point which is approximately constant for all angles.

When the term "moment coefficient" is used without qualification, it may always be assumed to refer to the pitching moment coefficient taken about the *quarter-chord* point. At zero lift there is an *aerodynamic couple* acting on an airfoil (Fig. 1–30). The moment of a couple is the same about any point in the plane of couple, therefore the moment coefficient at zero-lift is independent of the axis about which it is taken. It has been proved, further, that the lift due to the curvature of a wing acts at 50% of the chord, while the lift due to angle of attack acts at 25% of the chord. It therefore follows that the moment coefficient taken about the quarter-chord point should be constant for a given airfoil. An examination of the N.A.C.A. airfoils will show that the aerodynamic center is, for almost all airfoils, located at the quarter-chord point.

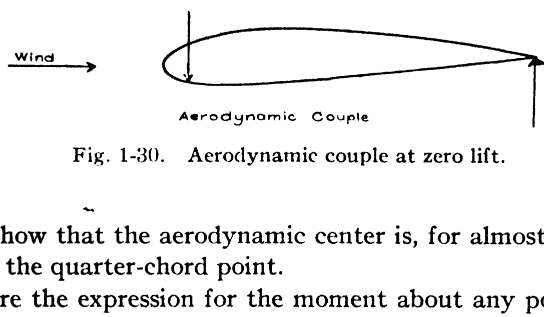


Fig. 1-30. Aerodynamic couple at zero lift.

Therefore the expression for the moment about any point located a.c. from the leading edge is, approximately, for small angles of attack:

$$M_{a.c.} = -C_L \, \rho/2 \, SV^2 \, (C.P. \times c - a.c.)$$

and the expression most used for the moment about the aerodynamic center (a.c.), which is practically always at the 25% chord from the leading edge, is

$$M_{0.25} = -C_L \, \rho/2 \, SV^2 \, (C.P. \times c - 0.25)$$

Example: Find the moment about the quarter-chord of an N.A.C.A. 23012 airfoil at a 4° angle of attack when the area is 200 sq ft, the chord is 6 ft, and the airspeed is 140 ft/sec.

Solution: From Fig. 1-19 when angle of attack is 4°

$$C_L = 0.39, \; C.P. = 26\%$$

Then the moment about the quarter-chord is

$$M_{0.25} = -0.39 \times \frac{0.002378}{2} \times 200 \times (140)^2 (0.26 \times 6 - 0.25)$$
$$= -2380 \text{ ft-lb}$$

PROBLEMS

1. Find the moment about the quarter-chord of a Clark Y airfoil at $-1°$ angle of attack when the area is 600 sq ft, the chord is 10 ft, and the airspeed is 100 mph.
2. Find the moment about the quarter-chord of a Clark Y airfoil at 7° angle of attack when the wing area is 150 sq ft, the chord is 5 ft, and the airspeed is 70 mph.
3. An airplane flying at 200 mph at 10,000 ft has an N.A.C.A. 23012 airfoil with 1350 sq ft of wing area. Find the moment about the aerodynamic center when the chord is 15 ft and angle of attack is 2°.
4. An airplane equipped with an N.A.C.A. M-6 airfoil with 384-sq ft wing area, chord 8 ft has for 8° angle of attack $C_L = 0.64$ and $C.P. = 0.25$. Find the moment about the quarter-chord if the airspeed is 85 mph.
5. Find the moment about the quarter-chord of an airplane flying at 128 mph at 12° angle of attack. The Clark Y wing has 433.5 sq ft of wing area and chord of 8.5 ft.

Power at Minimum $C_D/C_L^{3/2}$. The familiar expression for HP required may be transformed into an expression that does not include velocity.

$$HP_{req} = \frac{D \times V}{550}$$
$$= \frac{(C_D\, \rho/2\, SV^2)V}{550}$$
$$V^2 = \frac{W}{C_L\, \rho/2\, S}$$
$$V = \sqrt{\frac{W}{S}}\sqrt{\frac{1}{C_L\, \rho/2\, S}}$$

then
$$HP_{req} = \frac{1}{550} \times C_D\, \rho/2\, S \times \frac{W}{C_L\, \rho/2\, S}\sqrt{\frac{W}{S}}\sqrt{\frac{1}{C_L\, \rho/2\, S}}$$
$$= \frac{1}{550} \times \frac{1}{\sqrt{\rho/2}} \times W\sqrt{\frac{W}{S}} \times \frac{C_D}{C_L^{3/2}}$$

With reference to the above expression, a given airplane will require the least HP at the angle of attack at minimum value of $C_D/C_L^{3/2}$. It is interesting to examine Fig. 1-31 where the $C_L^{3/2}/C_D$ and C_L/C_D curves are plotted against angle of attack. The maximum C_L/C_D is the angle of attack where the least thrust force is re-

quired. The position of the maximum $C_L^{3/2}/C_D$ is different from that of the maximum C_L/C_D angle of attack, and is the angle at which the least power is required. The angle of attack for the maximum C_L/C_D is less than that for the $C_L^{3/2}/C_D$ and since this is true, the air-speed for the C_L/C_D must be greater. Power required is the product of force and velocity. The maximum C_L/C_D will require less thrust

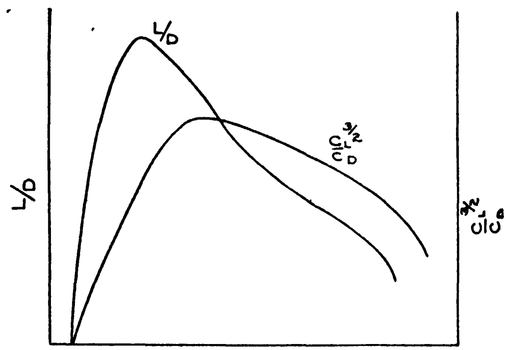


Fig. 1-31. Typical L/D and $C_L^{3/2}/C_D$ curves.

and allow greater velocity. Therefore, power required at the angle for maximum C_L/C_D will be greater than the angle of attack of maximum $C_L^{3/2}/C_D$. It is noted from the expression that at one angle of attack the power required varies directly as $W^{3/2}$, inversely as the square root of the wing area, and inversely as the square root of the density.

PROBLEMS

1. Plot a curve of angle of attack versus $C_L^{3/2}/C_D$ for N.A.C.A. 23012 airfoil.

2. An airplane with a gross weight of 3000 lb has a wing area of 250 sq ft. The wing is at an angle of attack for $C_L = 0.9$ and $C_D = 0.08$. What is the HP required by the wing?

3. An airplane with wing area of 300 sq ft and gross weight 3800 lb is flying at an angle of attack for $C_L = 1.25$ and $C_D = 0.12$. Find the HP required by the wing?

2
INDUCED DRAG

According to the previous explanations, there exists on an airfoil in flight a low-pressure area on the upper surface and a high-pressure area on the lower surface. In order to produce *lift momentum*, an equivalent amount of momentum in a downward direction must be given the surrounding air. The airfoil actually deflects a cylindrical stream of air with its diameter equal to the span of the wing. This necessarily results in a downward change of direction of the airflow producing what is called "downwash."

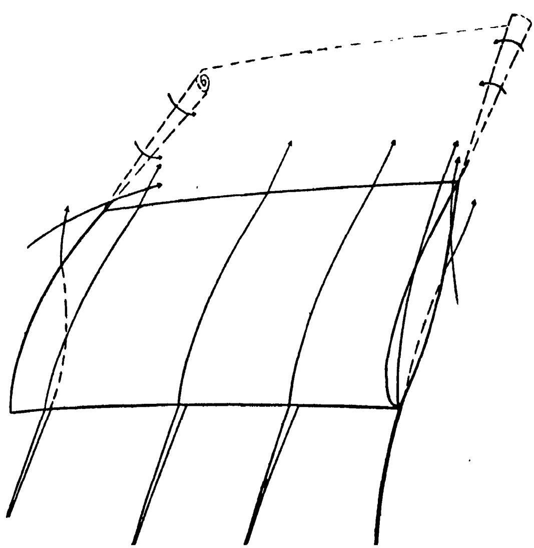


Fig. 2-1. Motion of air on upper and lower surface of wing.

It may be necessary at this time to examine the flow over an airfoil. It is seen that, due to an increase of pressure on the lower surface, there exists a tendency for the streamlines to move under the wing and outward toward the low-pressure area on the tip of the wing. Near the center of the span this outward motion is weaker than near the tip. This motion is shown in Fig. 2-1.

Due to the air coming up and inward over the tips, on the upper camber, there is an inward motion, which is strongest at the tips.

See Fig. 2-1. It is interesting to note that if an airplane wing had an *infinite aspect ratio* there would be no inward and outward motion and hence there would be no so-called "induced drag." This type of flow is called *two-dimensional.*

Vortices at Trailing Edge. With the movement of air as described above, it is evident that some reaction must occur. The streamlines coming under the lower side of the wing are directed backward and outward. On the upper surface there are backward, downward, and inward streamlines. The combination of these two motions at the trailing edge initiates a *vortex.* See Fig. 2-2. An examination of the figure shows the vortices on the left wing tip as counterclockwise. The outward and inward motions of the air tend to neutralize each other and this effect may be ignored. However, the downward components are considered very important.

There is no appreciable effect of wing tip vortices on a wing of infinite aspect ratio and the streamline flow is always in a vertical plane, parallel to the vertical plane of symmetry of the airplane. This type of flow is often referred to as *profile flow.*

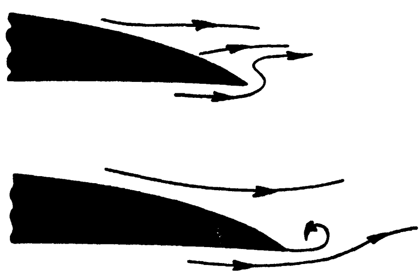


Fig. 2-2. Initiation of vortex at trailing edge of wing.

For convenience, the engineer generally refers the lift and drag components of the resultant air force to the direction of the free-air-stream velocity; that is, the lift is drawn perpendicular to the free stream and the drag is drawn parallel. However, in consideration of "downwash," it would be more accurate to refer these components to the mean direction of the airstream. Recognition of this idea leads to a conception of induced drag: Let Fig. 2-3 represent the flow of air past an airfoil. The air velocity relative to the airfoil is shown a short distance in front and is represented by the *free-stream velocity V.* Behind the airfoil is represented again the vector, V, of the free stream and an additional vector, V_1, called the *downwash velocity.* The velocity vector of the free stream, V, is equal to the downwash velocity, V_1 in magnitude, but the air is deflected through an angle

ϵ_r (called the *mean downwash angle*), the subscript r denoting the angle is expressed in radians. This means that, although the downwash velocity, V_1, is equal in magnitude to the free-stream velocity, V, an acceleration takes place because there is a change in direction. This change in direction is equal to ΔV, that is, it is equal to the difference between the velocity vectors V and V_1. By definition, force is equal to mass times acceleration. Since a mass of air, per unit time, is being deflected with a downward velocity of ΔV, a force is acting on the air. For every action there is an equal and opposite reaction; therefore the air must react with an equal and opposite force on whatever causes the deflection. Then, the force exerted on the airfoil is in the opposite direction to ΔV and is denoted by F (Fig. 2-3). In the figure, the force, F, is resolved into its components of lift and drag in the usual manner; where the lift (C_L) is perpendicular to the original free stream velocity V and the drag, parallel to the free stream velocity is labeled C_{Di} to indicate *induced drag*. Actually,

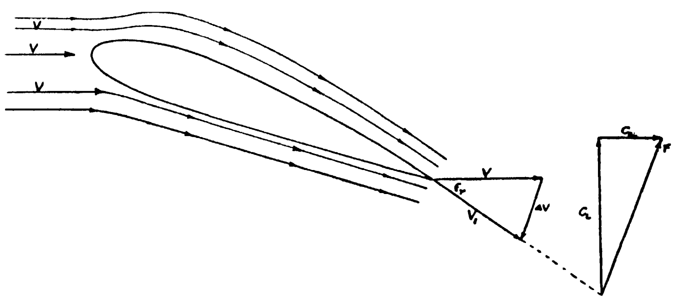


Fig. 2-3. Diagram of induced drag forces.

the total drag is greater than C_{Di} because of the fluid friction on the airfoil surface and the eddy currents behind the airfoil. The friction and eddy drag depend on the profile of the airfoil (*profile drag*). The total drag is considered the sum of the profile and induced drag:

$$C_D \text{ (Total Drag)} = C_{Di} \text{ (Induced Drag)} + C_{D_0} \text{ (Profile Drag)}$$

In other words, the term *induced drag* is considered the drag *induced* by lift.

While moving through the air, the wing imparts a motion to a large area of air. Although this disturbed mass of air has no definite size, it will be permissible to assume that a definite area is affected uniformly. This hypothetical area may be denoted by S_h and is a cross section of the airstream that is perpendicular to the motion of the airfoil.

The total mass of air affected by the airfoil in unit time is $\rho S_h V$. The downward momentum (mass × velocity) imparted to this

mass in unit time is $\rho S_h V \times V\epsilon_r$ where, by geometry $V\epsilon_r$ is considered to be equal to the downward velocity ΔV; it is measured in radians. It has been proved that the downward momentum in unit time is equal to one-half the lift.

Then $$\rho S_h V^2 \epsilon_r = L/2$$

therefore $$\epsilon_r = \frac{L}{2\rho S_h V^2}$$

If the swept area or affected area of air is a circle whose diameter is the span of the wing, Fig. 2-4,

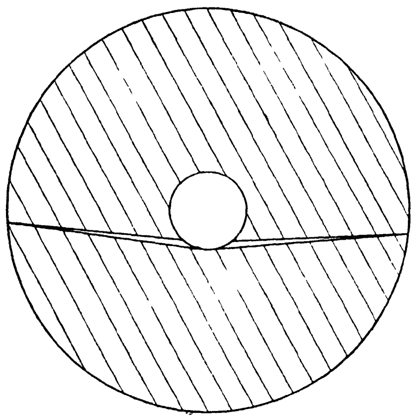


Fig 2-4. Area of air affected by wing.

then $$S_h = \frac{\pi b^2}{4}$$

$$\epsilon_r = \frac{L}{2\rho \frac{\pi b^2}{4} V^2}$$

$$= \frac{C_L\, \rho/2\, S V^2}{2\rho \frac{\pi b^2}{4} V^2}$$

$$= \frac{C_L S}{\pi b^2}$$

By definition: $$\text{Aspect Ratio} = \frac{\text{span}^2}{\text{area}} = \frac{b^2}{S}$$

then $$\epsilon_r = \frac{C_L}{\pi AR},$$

also from Fig. 2-3

$$C_{D_i} = C_L \epsilon_r = \frac{C_L^2}{\pi A R}$$

It must be kept in mind that the above expressions are not exactly correct for all types of wings. However, they introduce such a slight degree of error that, they are suitable for all practical work.

Example: A rectangular monoplane wing has a span of 43 ft and chord of 6 ft. What is the induced angle of attack and induced drag coefficient when $C_L = 0.8$? What is the induced drag when the velocity is 100 ft per sec?

Solution:

$$\epsilon \text{ (radians)} = \frac{C_L}{\pi A R}$$

$$\text{Aspect Ratio} = \frac{\text{span}}{\text{chord}} = \frac{43}{6} = 7.17$$

$$\epsilon^\circ = \frac{0.8}{3.1416 \times 7.17} \times 57.3$$

$$= 2.03^\circ$$

$$C_{D_i} = \frac{C_L^2}{\pi A R} = \frac{(0.8)^2}{3.1416 \times 7.17}$$

$$= 0.0284$$

$$D_i = C_{D_i}\, \rho/2\, S V^2$$

$$= 0.0284 \times 0.001189 \times 258 \times (100)^2$$

$$= 87.2 \text{ lb}$$

PROBLEMS

1. A rectangular monoplane wing has a span of 50 ft and a chord of 7 ft. Find the induced angle of attack and the induced drag coefficient when $C_L = 0.24$. If the velocity is 80 mph what will be the induced drag?

2. An airplane with a rectangular wing, 40-ft span, 6-ft chord, is flying at an angle of attack for $C_L = 1.2$. What is the induced angle of attack and the induced drag coefficient? If the velocity is 75 mph, find the induced drag.

3. An airplane flying level at 10,000 ft at a velocity of 120 mph has a gross weight of 8000 lb. The rectangular wing has a span of 90 ft and a chord of 10 ft. Find the induced angle of attack and the induced drag coefficient. What is the induced drag?

4. An airplane with a rectangular wing of aspect ratio 4.6 and chord of 7 ft is flying level at 150 ft per sec at 5000 ft. The gross weight is 3950 lb. Find: (a) the induced angle of attack; (b) the induced drag coefficient; (c) the induced drag.

5. An airplane with a Clark Y wing at 4° angle of attack with a span of 40 ft and chord of 6.8 ft is flying level at 100 mph at sea level.

Find: (a) the induced angle of attack; (b) the induced drag coefficient; (c) the induced drag.

Infinite Aspect Ratio Corrections. It is becoming customary to furnish characteristic curves for airfoils having an infinite aspect ratio. On a wing having an infinite aspect ratio, there are no wing-tip vortices. The wing with a finite aspect ratio has wing-tip vortices.

Up to now the geometric angle of attack has been referred to as the angle between the relative wind and the chord. However, since the theory of induced drag has been introduced, the more accurate *geometric* angle of attack may be considered the induced angle of attack, ϵ, plus the *effective angle of attack*, α_0 (Fig. 2–5).

$$\alpha \text{ (geometric)} = \epsilon + \alpha_0$$

With infinite aspect ratio, there is neither an induced angle nor induced drag. When the characteristics are given for an airfoil of infinite span, the angle of attack is identical with the effective angle of attack and the coefficient for drag is the profile drag coefficient. To

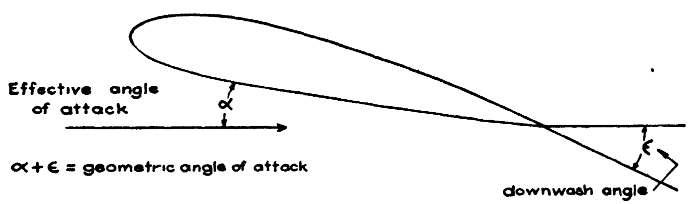


Fig. 2-5. Geometric angle of attack.

find the characteristics for a finite aspect ratio, it is necessary to add the effective angle of attack, α_0, to the induced angle of attack for that particular aspect ratio to give the total geometric angle of attack. Likewise, the profile drag coefficient is added to the induced drag coefficient, for the desired aspect ratio, to produce the total drag coefficient.

Example: For a particular airfoil of infinite aspect ratio at 6° angle of attack, the $C_L = 1.01$ and $C_{D_0} = 0.063$. Using these values, find the characteristics for a finite aspect ratio of 8.

Solution:

$$\alpha = \epsilon + \alpha_0$$

$$\epsilon \text{ (radians)} = \frac{C_L}{\pi AR}$$

$$\epsilon \text{ (degrees)} = \frac{C_L}{\pi AR} \times 57.3$$

$$\alpha = \frac{1.01 \times 57.3}{3.1416 \times 8} + 6°$$
$$= 2.3° + 6°$$
$$= 8.3°$$

$$C_D = \text{(total drag)} = C_{D_0} \text{ (profile drag)} + C_{Di} \text{ (induced drag)}$$

$$C_{Di} = \frac{C_L{}^2}{\pi AR}$$

$$C_D = 0.063 + \frac{(1.01)^2}{\pi \times 8}$$
$$= 0.063 + 0.046$$
$$= 0.109$$

That is to say, an airfoil with lift coefficient of 1.01 at 8.3° angle of attack will have a corresponding drag coefficient of 0.109 when corrected for an aspect ratio of 8.

Suppose the characteristics for an airfoil of one aspect ratio are known and it is desired to transform these values to the characteristics of an airfoil with an infinite aspect ratio.

Example: A certain airfoil with an aspect ratio of 6 has for a 3° angle of attack, $C_L = 0.3$ and $C_D = 0.04$. Find the corresponding characteristics for infinite aspect ratio.

Solution:

$$\alpha(AR\text{-}6) = \epsilon°(AR\text{-}6) + \alpha_0$$

$$3° = \left(\frac{0.3}{\pi 6} \times 57.3\right) + \alpha_0$$

$$\alpha_0 = 3° - 0.91°$$

then $\alpha_0 = 2.09°$ = angle of attack for infinite aspect ratio

when $C_L = 0.3$

and $C_D(AR\text{-}6) = C_{D_i}(AR\text{-}6) + C_{D_0}$

$$0.04 = \frac{(0.3)^2}{\pi \times 6} + C_{D_0}$$

$C_{D_0} = 0.04 - 0.0048$

$= 0.0352$ is drag coefficient for infinite aspect ratio when $C_L = 0.3$

PROBLEMS

1. An airfoil with an infinite aspect ratio has an effective angle of attack of 5°, $C_L = 0.8$ and $C_{D_0} = 0.02$. Find the corresponding values for a finite aspect ratio of 6.
2. An airfoil with aspect ratio of 6 has for its geometric angle of

attack of 7°, $C_L = 0.6$ and $C_D = 0.03$. Find the corresponding characteristics for an infinite aspect ratio.

3. Find the angle of attack and drag coefficient for an aspect ratio of 3, if the effective angle of attack for an infinite aspect ratio is 10°, $C_L = 1.3$ and $C_{D_0} = 0.023$.

4. Find the angle of attack and drag coefficient for an aspect ratio of 6, if the effective angle of attack for an infinite aspect ratio is 4°, $C_L = 0.55$ and $C_{D_0} = 0.01$.

5. An airfoil with aspect ratio of 6 has for its geometric angle of attack of 8°, $C_L = 0.9$ and $C_D = 0.059$. Find the drag coefficient and angle of attack for an infinite aspect ratio.

6. A Clark Y airfoil, aspect ratio 6, has for an angle of attack of 4°, $C_L = 0.65$ and $C_D = 0.034$. Find the drag coefficient and angle of attack for an infinite aspect ratio.

7. Find the angle of attack and drag coefficient for a Clark Y aspect ratio of 6, if the effective angle of attack for an infinite aspect ratio is 8°, $C_L = 1.2$ and $C_{D_0} = 0.019$.

8. An airfoil, aspect ratio 10, has for its geometric angle of attack of 3°, $C_L = 0.6$ and $C_D = 0.03$. Find the drag coefficient and angle of attack for an infinite aspect ratio.

9. An N.A.C.A. M-6 airfoil, aspect ratio 6, has for its geometric angle of attack of 5°, $C_L = 0.425$ and $C_D = 0.02$. Find the drag coefficient and angle of attack for an infinite aspect ratio.

10. Find the angle of attack and drag coefficient for an N.A.C.A. 23012 of aspect ratio 6, if the effective angle of attack for an infinite aspect ratio is 6°, $C_L = 0.75$ and $C_{D_0} = 0.01$.

Formerly, all the available data on airfoils were given for an aspect ratio of 6 but in modern aircraft design it is now very rare to find an airplane of this value of aspect ratio. It follows then that the most plausible solution would be to give the characteristics of the airfoil for an infinite aspect ratio, and let the designer transpose these values to the desired aspect ratio. In some cases, however, this is not feasible, and there may be a desire to transform the characteristics of one aspect ratio to the corresponding aspect ratio desired. In this case the following method is described.

Example: An airfoil, with aspect ratio of 6, at an angle of attack of 4°, has a $C_L = 0.37$ and $C_D = 0.017$. Find the angle of attack and C_D that will correspond to the C_L of 0.37 of the same airfoil if the aspect ratio is 8.

Solution:

$$C_{D_6} - C_{D_8} = C_{D_{i_6}} - C_{D_{i_8}}$$

where C_{D_6} = total drag of airfoil AR-6
C_{D_8} = total drag of airfoil AR-8

$$C_{D_{i_6}} = \frac{C_L^2}{\pi 6} = \text{induced drag of airfoil AR-6}$$

$$C_{D_{i_8}} = \frac{C_L^2}{\pi 8} = \text{induced drag of airfoil AR-8}$$

It will be noted that profile drag was omitted because of independence of aspect ratio.

$$C_{D_6} - C_{D_8} = \frac{C_L^2}{\pi 6} - \frac{C_L^2}{\pi 8}$$

$$= \frac{C_L^2}{\pi}\left(\frac{1}{6} - \frac{1}{8}\right)$$

$$= \frac{(0.37)^2}{3.1416}\left(\frac{1}{6} - \frac{1}{8}\right)$$

$$= 0.001813$$

$$0.017 - 0.001813 = 0.01518$$

then C_D for AR-8 $= 0.01518$

also $\alpha_6 - \alpha_8 = \epsilon_6^\circ - \epsilon_8^\circ$

where α_6 = total angle of attack for airfoil with AR-6

α_8 = total angle of attack for airfoil with AR-8

$\epsilon_6^\circ = \frac{C_L}{\pi 6} \times 57.3$ = induced angle of attack for airfoil with AR-6

$\epsilon_8^\circ = \frac{C_L}{\pi 8} \times 57.3$ = induced angle of attack for airfoil with AR-8

then
$$\alpha_6 - \alpha_8 = \frac{0.37 \times 57.3}{\pi 6} - \frac{0.37 \times 57.3}{\pi 8}$$

$$\alpha_6 - \alpha_8 = \frac{0.37 \times 57.3}{\pi}\left(\frac{1}{6} - \frac{1}{8}\right)$$

$$= 18.24 \times 0.37\left(\frac{1}{6} - \frac{1}{8}\right)$$

$$= 0.281^\circ$$

then $4^\circ - 0.281^\circ = 3.719^\circ$ = angle of attack for airfoil with aspect ratio 8

Problems

1. An airfoil, with aspect ratio of 6, at an angle of attack of 7°, has a $C_L = 0.68$ and $C_D = 0.03$. Find the angle of attack and C_D that will correspond to the C_L of 0.68 of the same airfoil if the aspect ratio is 12.

2. An airfoil, with aspect ratio of 8, has for an angle of attack of 2°, $C_L = 0.3$ and $C_D = 0.01$. Find the corresponding angle of attack and C_D for an aspect ratio of 3.

3. An airfoil section, aspect ratio 6, for 9° angle of attack has a lift coefficient of 0.95 and a drag coefficient of 0.07. Find the corresponding angle of attack and C_D for an aspect ratio of 9.7.

4. Find the corresponding angle of attack and C_D for an aspect ratio of 4 if an airfoil, aspect ratio 6, has for 12° angle of attack, $C_L = 1.25$ and $C_D = 0.56$.

5. Calculate, for all angles of attack, the C_L and C_D for Clark Y of aspect ratio 10 and plot on same graph the C_L and C_D for aspect ratio 6.

Span Loading. At this point it is necessary to distinguish between two conditions:

1. Induced drag coefficient depends upon effective aspect ratio.
2. Induced drag is dependent upon span loading.

Span loading is the gross weight of the airplane divided by the span. This dependency of the induced drag coefficient can best be understood by the following expressions:

$$D_i = C_{D_i}\, \rho/2\, SV^2$$

and
$$C_{D_i} = \frac{C_L{}^2 S}{\pi b^2}$$

then
$$D_i = \frac{(C_L\, \rho/2\, SV^2)^2}{\pi\, \rho/2\, V^2 b^2}$$

$$= \frac{(L)^2}{\pi q b^2}$$

where $q = \rho/2\ V^2$

Then for level flight the lift equals weight:

$$D_i = \left(\frac{W}{b}\right)^2 \times \frac{1}{\pi q}$$

$$W/b = \text{span loading}$$

Therefore, for any given lift coefficient, the induced drag coefficient depends upon the effective aspect ratio, but the induced drag at any given speed depends upon the square of the span loading. With a given lift, the span should be as large as otherwise practical in order to obtain the best climb and ceiling, since the induced drag is most important in securing good climb and ceiling. The wing area is generally determined by the maximum lift coefficient and the stalling speed desired.

When the wing area has been determined a large span will give a large aspect ratio, since the aspect ratio is equal to the square of the span divided by the area. Hence to say that a low span loading is desirable is another way of saying that a large aspect ratio is desirable. The aerodynamicist uses one or other criterion whichever may be more convenient at a particular stage of his work.

An elliptical loading (see Fig. 2–6) along the span of the wing is desirable because for this type of loading there is obtained the least induced drag for a given span loading. For elliptical loading the induced angle also has its smallest value. Designers place special em-

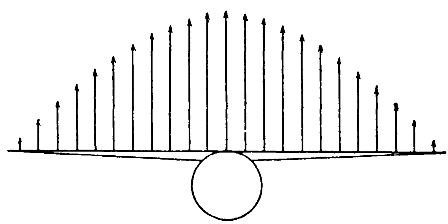


Fig. 2-6. Elliptical air loading on elliptical planform.

phasis on securing elliptical loading for planes designed for long-range cruising. The simplest way of securing elliptical loading is to give the wing elliptical plan form, use the same profile throughout and place each element at the same angle of attack.

PROBLEMS

1. An airplane weighs 2000 lb and has a wing span of 48 ft. Find the induced drag at standard conditions if the airplane is flying level

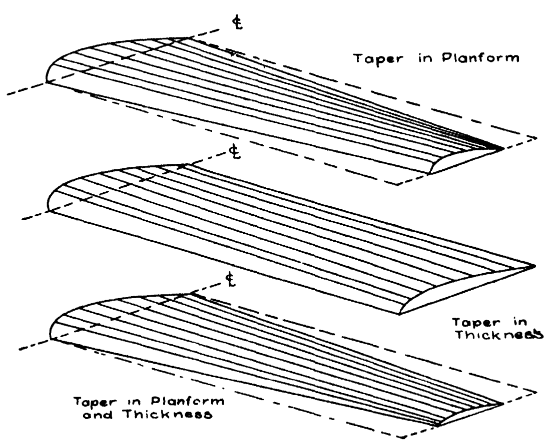


Fig. 2-7. Three methods of tapering a wing.

at 85 mph. What is the horsepower required to overcome the induced drag?

2. Find the induced drag of a level-flying, 4000-lb airplane with wing span of 65 ft at 10,000 ft at 100 mph. What is the HP required to overcome the induced drag at this altitude?

3. Find the induced drag of an airplane having a wing span of 100 ft and a gross weight of 8000 lb at sea level. The velocity is 120 mph. What is the HP required to overcome the induced drag?
4. An airplane, flying at 90 mph, has a gross weight of 1500 lb and wing span of 22 ft. What is the induced drag in level flight at sea level? With the gross weight the same but the span increased by 15 ft what is the induced drag?
5. Find the induced drag of an airplane flying at 200 mph at 5000 ft with gross weight of 3000 lb and wing span of 50 ft. What is the HP required to overcome the induced drag?

Tapering the Wings. Experience in design has taught that there are several aerodynamic advantages in tapering an airfoil from the *root section* (or *center section* as it is sometimes referred to). In actual

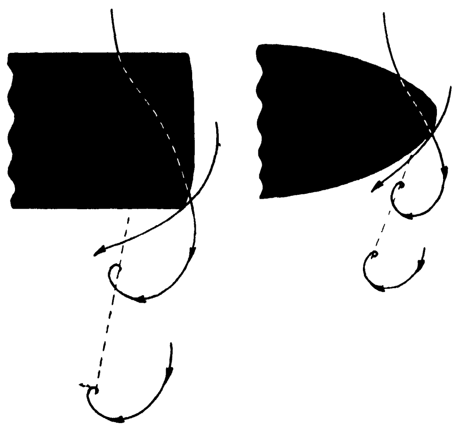


Fig. 2-8. Reduction of wing tip vortices by taper.

practice the aerodynamic advantages may be offset by the fact that fabrication of this kind increases the cost of production so greatly that the advantages of tapering the wings are curtailed by the increased cost of construction. There are, generally speaking, three ways of tapering an airfoil (see Fig. 2-7):

1. Tapering the wing in planform by reducing the chord from root section to tip. Although the airfoil sections remain the same, the actual thickness of the wing is decreased, because the values of upper and lower camber are unchanged.
2. Taper in thickness only. The values of upper and lower camber are reduced by a constant percentage from the root section to the tip. The chord, however, remains the same throughout.
3. Tapering the wing in both planform and thickness.

Results of Giving an Airfoil Taper in Planform. It has been noted in

wind-tunnel tests that the resultant force, called the *lateral center-of-pressure,* of a wing tapered in planform was closer to the center line. This fact was found true at every angle of attack and in this respect the planform tapered wing is superior to the rectangular-form wing. Other reasons are as follows:

1. Due to the tapering of the wing tips, the induced wing tip vortices are considerably reduced and, in turn, the induced drag is reduced. (Fig. 2-8.)
2. Such an airfoil should require a lighter structure for the same strength.
3. In flight tests this type of wing has been easier to control laterally than a rectangular wing.

As has been indicated in N.A.C.A. Technical Report No. 152, the taper in planform is superior to the rectangular planform in that it gives:

1. A higher maximum L/D.
2. A higher maximum lift coefficient.
3. Higher values of L/D at the larger values of the C_L, giving a wing that is better for climb.

Although the difference is not too great, it should be noted that the C.P. travel is slightly larger for the tapered airfoil.

Results of Giving Airfoil Taper in Thickness Only. It was found that the lateral center of pressure, that is, the resultant force spanwise, gave results similar to the taper in planform in that they both have the C.P. closer to the center section. The following effects of an airfoil which is rectangular in planform and tapered to 25% thickness at the tip were found (N.A.C.A. Technical Report No. 152):

1. The minimum drag decreases.
2. The maximum lift decreases.
3. The maximum L/D increases.
4. The L/D increases at small values of the lift coefficient.
5. The C.P. travel between given values of the lift coefficient is not great.

Results of Use of Taper in Planform and Thickness. A complete account on some airfoils of this type may be found in N.A.C.A. Technical Report No. 152. The results, however, give the advantages of both types and by careful designing, slightly better results may be obtained than either of the two types. The taper must not be as great as 25% thickness at the tip because of too flexible a wing.

3
AIRFOIL SELECTION

The selection of a wing section constitutes a decision of great responsibility. It is not always practicable to continue to use the same wing section as has been used before or used by a competitor in a similar design. A change in the structural design may tend to retard progress and bar a most promising way to improvement. In the selection of each new design, it is best to determine whether the airfoil section that appears at first is better suited or whether a new section might bring about better results. There are many who advocate selection of airfoils on a strictly mathematical basis; their ideas are somewhat too specific and not broad enough. Actually, the performance of the airplane does not, and should not, depend on the wing section alone. The complete design should be analyzed, involving the interference between the other parts of the plane and the wing. The simplest mathematics may be used to effect an intelligent choice and hardly any difficult computation is needed.

Among the so-called wing sections investigated in the wind tunnels, at least 90 percent can be condemned at first sight, either because of the contours of the airfoil or on account of its aerodynamic characteristics. The first requirement of a section is that it fits the structural design. If the airplane is to be of *cantilever design,* this particular section considerably limits the number of prospective wing sections. The engineer has a rather clear idea of the depths that the spars must have and also he knows the definite positions for them.

General Considerations. The aerodynamic properties actually classify themselves into three groups:

(a) General aerodynamic behaviors
(b) Stability
(c) Performance

It has been found that some sections, while showing favorable aerodynamic properties, are extremely sensitive to small change of their shape. It naturally follows that such an airfoil will be as sensitive to interference of adjacent parts of the airplane or of protruding fittings. This condition is usually discernible from irregularities in the variation of the air forces. For example: The curves showing the relation between lift and drag, or lift and angle of attack, may have

bumps or even steps. The pressure distribution also shows an irregular character.

These airfoil sections with irregular characteristics should be avoided because their choice involves a great risk. However, the irregularities near the smaller angles of attack are less dangerous than those at higher angles of attack.

The C_L curve beyond the maximum value is another very important consideration. The lift coefficient should maintain its value over a wide range of angle of attack. If it falls off, it should fall off gently and continuously, never by a steep and sudden, large step. An airplane equipped with this type of airfoil will tend to fall off on one wing at low speed or will tend to have an unnatural spin characteristic, or perhaps display some other violent behavior.

Desired Airfoil Performance. The designer usually selects as a basis for his design the engine having the power, performance, dimensions, and installed weight which he desires. His next problem is to choose the airfoil which he will use. Although it is recognized that in every design there are certain sacrifices that must be made, the engineer wants an airfoil that will give:

(a) The lowest landing speed for a given wing area;
(b) The greatest possible high speed;
(c) The best rate of climb;
(d) Minimum structural weight;
(e) Low parasite resistance.

Structural Requirements. If the designer should be limited to the design of an internally braced monoplane, for instance, he must select an airfoil having the dimensions that can accommodate the required structure. A designer had best make a tentative selection of several airfoil sections that seem to meet the aerodynamic requirements. Those selected should be compared in more detail, and then more consideration should be given to structural requirements. The final selection may well be the section that affords the best compromise between aerodynamic and structural requirements.

The use of airplanes with more than one wing is rapidly fading out of modern design trends because of wing interference, poor visibility, more complicated design, more drag and other limiting factors too numerous to mention. However, there still exist quite a number of successful biplanes that are used primarily as training planes. Since, in training planes, speed is not a requirement whereas stability and low landing speed are, it is evident that these requirements are well met in the design of the biplane. Actually, one of the main reasons for the biplane is that in airplane design the required wing area is governed by the total weight of the airplane. Most frequently land-

ing speed is the limiting quality, and this is dependent upon the weight of the airplane, the wing area, and the maximum lift coefficient of the airfoil. With these problems confronting the engineer a few years ago, the trend to heavier aircraft meant one of two things: (1) to make a huge monoplane wing that must be heavily reinforced outside, or (2) to get the required amount of wing area by incorporation of two or more wings. At that time, however, aspect ratio and heavy structural requirements limited construction.

In the past two years, science has brought improvements in materials and technique of construction, leading to modern aircraft design. Modern engineering has developed an efficient but more costly type of design which has made possible the internally braced (cantilever) monoplanes of large span.

Comparing Airfoils. Before the preliminary design is carried out, the designer should select two or more sections that best meet his particular requirements. After close inspection he should be able to narrow his choice down to two types. However, the method of placing two sets of characteristic curves together and comparing them visually is certainly not satisfactory.

In making a comparison of airfoil data from wind-tunnel tests, it is of primary importance that the models tested be of the same size and that the test be run at the same airspeed. Also, because of variance in wind tunnels it is desirable to have both sets of characteristic curves from the same tunnel. As has been explained before, the aspect ratio of the two should be the same and also the two Reynolds Numbers should be as close as possible to each other. Geometrical similarity is of utmost importance in comparison of any two sets of characteristics.

Tabulation and Discussion of Airfoil Data. Many methods of comparing airfoil sections have been devised and each one has its limitations; actually the final decision is usually made by comparing the calculated performance of the complete airplane.

In this chapter one method of comparison is shown that should give a fairly good idea of what may be expected of the complete airplane. The data for two airfoils have been placed side by side so that a direct comparison of each wing may be made easily.

TABLE II. COMPARISON OF AIRFOILS

Göttingen 398 and 436 Airfoils

1.	Airfoil Sections—Göttingen	398	436
2.	Date Tested	3/18/31	3/27/31
3.	Tested at	N.A.C.A.	N.A.C.A.
4.	Aspect Ratio	6	6
5.	Reynolds Number (RN)	3,070,000	3,050,000
6.	Pressure (Standard Atmosphere)	20.3	20.5

7.	Size of Model	30″ × 5″	30″ × 5″
8.	Angle of Attack $C_L = 0$	−6°	−4.8°
9.	$C_{L\,max}$	1.572	1.576
10.	Angle of Attack $C_{L\,max}$	18.5°	18.7°
11.	$C_{D\,min}$	0.0112	0.0099
12.	Angle of Attack $C_{D\,min}$	−5.2°	−3.7°
13.	Max L/D	20.10	21.5
14.	Angle of Attack at Max L/D	0.5°	1°
15.	L/D at $^2/_3\ C_{L\,max}$	13.5	14.0
	at $^1/_2\ C_{L\,max}$	16.5	17.2
	at $^1/_4\ C_{L\,max}$	19.6	21.2
	at $^1/_6\ C_{L\,max}$	16.5	18.5
	at $^1/_8\ C_{L\,max}$	14.5	16.00
16.	$\frac{C_L{}^3/_2}{C_D}$ at 6°	14.9	15.20
	at 8°	14.10	14.85
	at 12°	12.72	13.55
17.	Max. Forward C.P. at $C_{L\,max}$	31.3%	29.6%
18.	Max. Rearward C.P. at $C_{D\,min}$	77.2%	59.4%
19.	C_{MLE} at 0°	−0.189	−0.143
20.	Thickness at 10%	11.07	8.93% Chord
	at 15%	12.50	10.16 Chord
	at 60%	10.30	8.28% Chord
	at 70%	8.18	6.60% Chord

After the data have been prepared, the next step is for the designer to take each item and consider its possibilities. Naturally, one must keep in mind the type of airplane he is to make and choose the one airfoil that will be the best all-around performer.

Item 1. After fairly careful consideration of shape and general appeal of several airfoils, the designer finally narrows his selection to two. In this particular case the Göttingen 398 and 436 have been arranged for discussion.

Items 2 and 3. The next step, and a very important one, is consideration of date of the wind-tunnel testing and the place. Because of the many different wind tunnels and their peculiar characteristics, sometimes called "wall interference," it is very important that the airfoils to be compared have been tested in the same wind tunnel. The two Göttingen airfoils were both tested by the N.A.C.A.

Item 4. It has been explained in the previous chapter that the characteristics of one airfoil changed appreciably with a change in aspect ratio. Since the induced drag decreases with an increase in the aspect ratio, it is mandatory, for a true comparison, that the two sections, unless corrected for infinite aspect ratio, be of the same aspect ratio. In this case, the aspect ratio of six is common to both airfoils.

Item 5. Not only is it desirable for the two airfoils to be tested in the same wind tunnel, but the RN should also be as near the same as possible. Only when the airflow of two airfoils is geometrically similar can a valid comparison be made. The RN of the 398 is just slightly greater than that of the 436, and although this is not the ideal

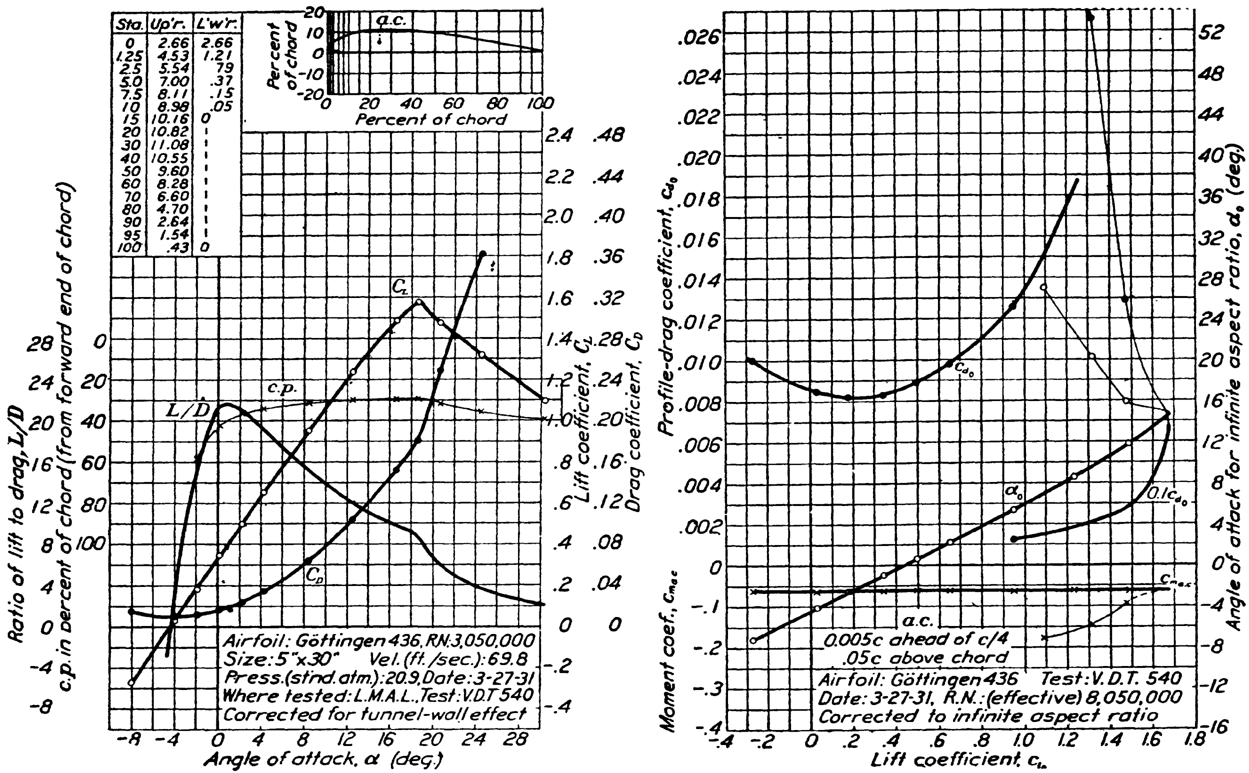


Fig. 3-1. Göttingen 436 airfoil, N.A.C.A.

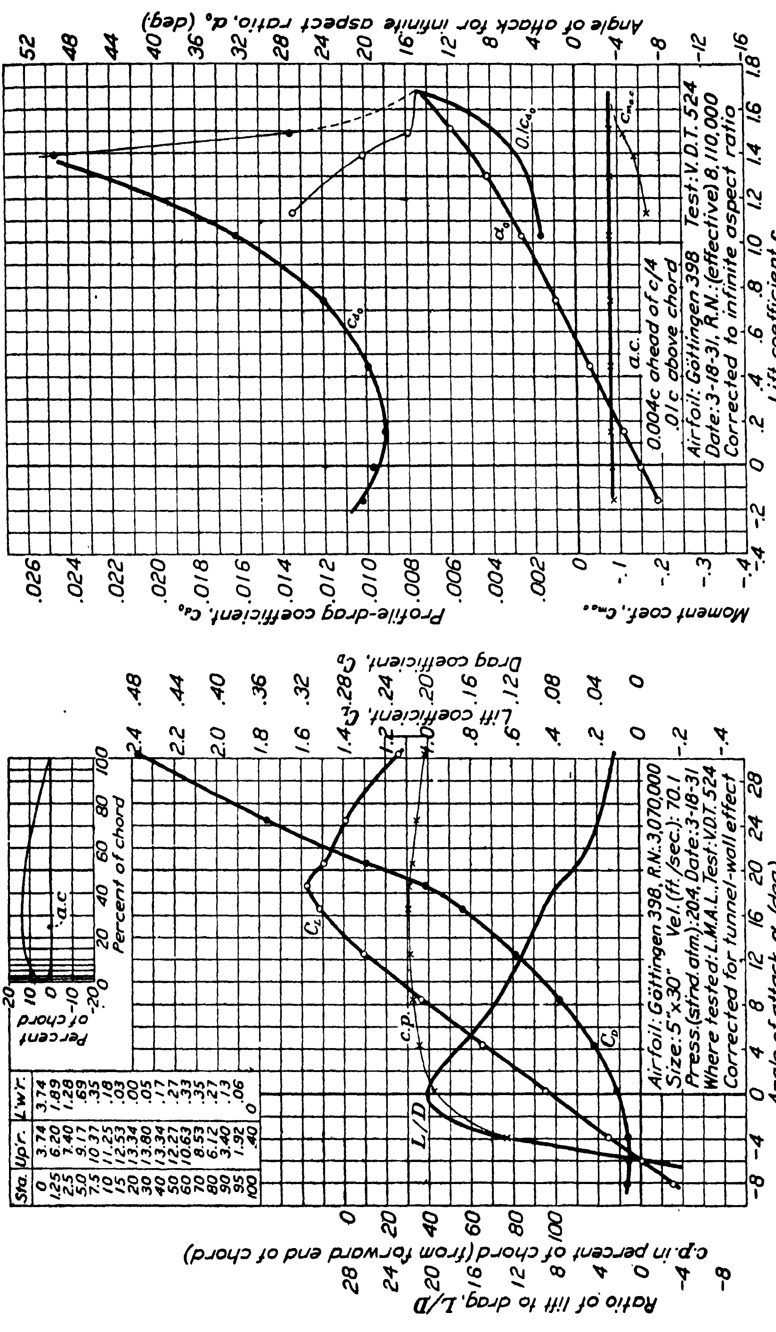


Fig. 3-2. Göttingen 398 airfoil, N.A.C.A.

situation, the difference is not enough to incorporate any appreciable degree of error. The student should be aware, however, that any great departure from similar RN's should be avoided because of the possibility of change in the characteristic curves, which would seriously affect the integrity of the comparison.

Item 6. The pressure in atmospheres is directly related to the RN through the change in density due to variable pressure. This is seen from the equation

$$RN = \frac{\rho V L}{\mu} \qquad \text{where } \begin{aligned} \rho &= \text{the density} \\ V &= \text{velocity (ft/sec)} \\ \mu &= \text{the coefficient of viscosity} \\ L &= \text{the length of the chord} \end{aligned}$$

Item 7. The size of the model which also affects the RN as seen in the above expression is important in assuring similarity between the two sections to be compared. Both sections above have the same dimensions.

Item 8. The angle of attack for $C_L = 0$ indicates the angle of attack for the wing when it is in a vertical dive. The type of plane will, of course, decide whether this particular item is of any value. For example, in a dive bomber, this angle of attack would be of great importance and the airfoil with the lesser negative angle of attack probably would be more desirable for that particular feature. However, it must be kept in mind continually that no one specific item should be the deciding feature but the broader all-around merits should be the resulting choice. The accompanying table shows that the Göttingen 436 with the $-4.8°$ angle of attack differs from the 398 by 1.2°.

Item 9. The maximum lift coefficient ($C_{L\,\max}$) is used in the lift formula to find required wing area

$$S = \frac{\text{lift} = \text{weight}}{C_{L\,\max}\ \rho/2\ V^2}$$

The maximum lift coefficient may also be used to find the landing speed, since the airplane usually is brought to the stalling point as the wheels touch the ground.

The Göttingen 436 is only slightly superior to the 398 and, in this case, would give approximately the same values. If either of the two airfoils were designed with *flaps*, the maximum lift coefficient with flaps extended would be greatly increased over the maximum C_L of the plain wing. The N.A.C.A. has published many characteristic curves of airfoils with flaps extended. The designer may then, by the use of these characteristic curves and the above formula determine

the amount of flap deflection and design considerations that best meet his requirements.

Item 10. The angle of attack for $C_{L\,\max}$ is important in determining the angle of the airplane for a three-point landing. Here again, the type of design will be the deciding influence on the choice. For instance, the use of the tricycle landing gear may make one airfoil superior to the other for that particular item. Good visibility in landing may be of prime importance to one design and of no particular value to another.

The angle of attack of $C_{L\,\max}$ for the Göttingen sections shows that the 436 has a greater angle than the 398 by 0.2°. If the particular type of plane is to have a tricycle landing gear the 398 may be superior but the 436 may be superior if the conventional type gear is employed. For items of this nature, it is not practicable to say which of the two is superior because of the difference in design.

Item 11. The *minimum drag coefficient* ($C_{D\,\min}$) indicates the section that will give the best top speed. The Göttingen 436 is slightly superior to the 398 in this case.

Item 12. The angle of attack for the minimum drag coefficient indicates the angle of incidence of the wing. This is of particular importance in the take-off attitude of the airplane and also the general appeal of the design. The Göttingen 436 again seems to indicate a more desirable characteristic since the angle of incidence required is smaller than that of the 398.

Items 13 and 14. The maximum *lift-drag ratio* is indicative of the best cruising speed and the angle of attack at maximum L/D shows the angle for cruising. The above comparison gives the Göttingen 436 the edge over the 398 and there is only 0.5° difference in the angles of attack.

Item 15. The values of L/D at $^2/_3\ C_{L\,\max}$ indicates which section will give the better climb-and-cruising characteristics. For these values the Göttingen 436 is greatly superior to the 398.

The value of L/D for $^1/_4\ C_{L\,\max}$ indicates the airfoil that will be the best suited for transport work. It has been found by experience that the heavier and larger airplanes operate at an angle of attack that is approximately $^1/_4$ of the angle of attack for $C_{L\,\max}$. It can be seen that, in this case, the Göttingen 436 is more desirable. The value of L/D at $^1/_6\ C_{L\,\max}$ indicates the one best suited for moderately high-speed airplanes because this class of airplane has been found to work usually at the angle of attack of $^1/_6\ C_{L\,\max}$. The 436 again shows it superiority. Lastly, the airplane of the pursuit and fighter type works at an angle of attack of approximately $^1/_8\ C_{L\,\max}$. The maximum value,

then, of the L/D at $^1/_8\ C_{L\ max}$ would show the airfoil best suited for high speed. The Göttingen 436 is again superior in this particular case.

Look at the two airfoils separately and determine the type of plane each is best suited for. An inspection of the maximum L/D at the different values of $C_{L\ max}$ indicates that the Göttingen 398 would be best suited for transport work. The Göttingen 436 also indicates better values as a transport type airplane.

Item 16. The ability of an airplane to climb and the maximum ceiling it can obtain are chiefly dependent on the difference between the power required to maintain level flight and the power the engine unit can supply. The expression for horsepower required is:

$$HP_{req} = \frac{1}{550} \times \frac{1}{\sqrt{\rho/2}} \times W\sqrt{\frac{W}{S}} \times \frac{C_D}{C_L{}^{3/2}}$$

The weight and wing area and power available are constant for the airplane; therefore, it can be seen, by this expression, that the airfoil which has the maximum value of $C_L{}^{3/2}/C_D$ will probably give the best climb and ceiling. The maximum value of the expression will usually occur at angles of attack between 6° and 12°. It is sufficient to make computations for a few angles of attack and from these determine the maximum values. For the Göttingen 398 and 436, the maximum values were taken for angles of attack of 6°, 8°, and 12°. The Göttingen 436 is shown to be superior at all angles of attack.

Items 17 and 18. The maximum forward position of the C.P. at $C_{L\ max}$ is very important in the stability of the airplane. The designer locates the wings of the airplane so that the maximum forward position of C.P. is vertical with the most rearward position of center of gravity. Also the maximum rearward position of the C.P. at the $C_{D\ min}$ will indicate the airfoil that has the least travel. It is always desirable to have the least possible travel of the C.P. between the high and low speeds because of structural and balance considerations.

The C.P. travel of the 436 seems to be much less than the 398 and this means that the 436 will not be as difficult to balance in the flight range. Also, the 436 will not require so heavy a structure as the 398. Balance and structural weight are highly important in the final flight characteristics.

Item 19. When an airfoil has been tested in the wind tunnel, the moments about the leading edge, at different angles of attack, are determined. The moment about the leading edge may be taken at any arbitrary angle of attack; the important consideration is that it be the same angle of attack for both sections. It can be seen that at an angle of attack of zero degrees the Göttingen 398 has a greater mo-

ment coefficient and since the moment about the leading edge is expressed

$$M_{LE} = C_{MLE} c\ \rho/2\ SV^2$$

the stresses on the spars would be greater for the 398. The requirement for greater strength would mean heavier structure and this in turn would mean an increased wing loading, more power required, faster landing speeds or greater wing area, etc. If neither of the sections gives an excessive moment at this arbitrary angle of attack, it may be desirable to plot moment curves for both airfoils and compare them for the complete flight range. In other words, by the approximate method the moment coefficients may be found by the formula

$$C_{MLE} = -(C.P.)(C_L)$$

The moments can be plotted against the angles of attack in the flight range, *i.e.*, the angles of attack from, say, $-5°$ to $20°$. In this manner, the moments of both airfoils may be compared point for point. At negative angles of attack some airfoils are found to have the C.P. curve discontinued. Such is the case of the two Göttingen sections. This usually indicates that, near the angle of attack for zero lift, the C.P. position is off the trailing edge and could not be determined accurately. At the angle for zero lift, which is usually the attitude for a vertical dive, the resultant forces perpendicular to the airstream equal zero; but there is a down-load on the leading edge and an up-load on the trailing edge. This terrific twisting moment on the airfoil at the angle for zero lift represents the most critical torsional stress that the airplane spars will be subjected to.

It was stated that the 398 gave a greater moment at the angle of attack of zero degrees. If the C.P. curves for both airfoils are extrapolated, it looks as though they both will be located off the trailing edge.

Item 20. Here the thicknesses of the two sections have been tabulated to show which of the two can provide the greater depth for the front and rear spars. With modern airplane design tending toward "aerodynamic cleanliness," such as cantilever wing, where all the bracing is hidden inside the wing, the deeper the airfoil section the stronger the spars can be made.

The first two points, 10 percent and 15 percent, are the usual limits of location for the front spar, and the other two points, 60 percent and 70 percent, are the usual limits of the rear spar location. The spar depths available are approximately 50 percent greater for the 398.

4
PARASITE DRAG

The necessity for keeping down the drag of an aircraft is evident but the actual means of accomplishing this is not at all obvious. Knowledge of this subject is very limited and what little is known about drag has been learned from long experience.

Everyone is familiar with the one universal method of reducing air resistance: *streamlining.* The idea of streamlining is to mold and form the parts so as to give flowing air the easiest path around them with the least energy absorption.

When an object is moved through the air, resistance is encountered. The force that the air exerts on an object has a component acting parallel to the direction of the body with respect to the air. This component is called *drag.* It must be noted that the air resistance of all exposed parts of an airplane, other than the induced drag of the wings, is the parasite drag. The total drag of the airplane includes the profile drag of the wings, and the individual drag of such parts as wires, braces, struts, fuselage, landing gear, fittings, etc.

Drag is the sum of two kinds of air resistance:

1. Skin friction, or viscous resistance; and
2. Turbulence, or eddy-forming resistance.

The *skin friction* of a body, referred to earlier as *shearing stress* occurs whenever a viscous fluid moves over the surface of a solid body or when there is a relative motion or shearing action between the adjacent layers of the fluid itself. Turbulency is developed whenever there is a breakdown in the smooth streamline flow of the fluid. This causes *eddying* disturbances in the fluid as it moves past the object. Even the slightest variation from parallel flow in the streamlines tends to set up turbulent flow around the body; therefore, the skin frictional resistance never exists alone. Actually when a real fluid flows by a solid body, the flow will never be truly laminar. However, a portion of the flow about a body may be laminar and produce skin-frictional resistance only, but even with the best of streamline shapes turbulence is always present, and lends the greater part of the total drag.

Parasite Drag Coefficient may be defined by the relation:

$$C_{D_p} = C_D - \frac{C_L^2}{\pi A R}$$

In other words, parasite resistance is considered as the total drag minus the induced drag.

The actual parasite drag of an airplane can be used in connection with this coefficient by the usual formula:

$$D_p = C_{D_p} \times qS \quad \text{where } q = \rho/2\ V^2$$
$$S = \text{wing area as usually defined}$$

Many other coefficients have been used for the representation of the parasite drag of airplanes and their component parts. Because the scope of this book does not allow space for all the different types of formulas and data, it will be necessary only to point out some of the main types and examples. The student who desires further information should secure the many N.A.C.A. Technical Reports which carry on in greater detail.

Equivalent Flat-Plate Area (E.F.P.A.). Consideration of the general formula for resistance of different bodies at certain given speeds, atmospheric conditions, etc., has enabled the designer to arrive at some very important conclusions as to airplane performance. Remembering the main formula for air resistance as

Resistance in pounds = 0.00152 × area in sq ft × (velocity in ft per sec)^2

If the parasite resistance of the airplane is computed by summing up computations of data of various parts, the total resistance will be given as a certain number of pounds at a given velocity. With these data and the above formula, the so-called *equivalent flat-plate area* (E.F.P.A.) may be obtained.

For example, suppose the total value of resistance was found to be 400 lb at 100 mph, the E.F.P.A. will be

$$A, \text{ (E.F.P.A.)} = \frac{400}{0.00152 \times \left(100 \times \frac{44}{30}\right)^2} = 12.25 \text{ sq ft}$$

It is always advisable, however, to get resistance data from the actual, complete model. After a flight test has been made on an airplane, the total drag at high speed may be determined by the characteristics of the engine-propeller combination, since the total drag will be equal to the thrust. Then, as has been suggested, by computation from the wing data (induced drag) the parasite resistance may be determined. If this procedure is carried out for a large number of airplanes, a designer may, by careful comparison of his new design

with a similar plane already built, make a fairly close guess as to the probable E.F.P.A. of the new airplane. It will be shown later that this procedure may be followed for fuselages, etc., and the results will be as accurate, if not more so, than the summation of the drag of the parts.

PROBLEMS

1. An airplane whose design speed is 300 mph has an E.F.P.A. of 4.3 sq ft. Find the total drag at the design speed.
2. Find the E.F.P.A. of an airplane if the drag is 300 lb at 120 mph.
3. An airplane is flying at an angle of attack for $C_L = 1.2$ and $C_D = 0.10$. If the wing area is 300 sq ft, aspect ratio 8.2, find the actual parasite drag if the airspeed is 175 ft/sec.
4. Find the parasite drag coefficient for an airplane whose wing of aspect ratio 12 is flying at a value of $C_L = 0.5$ and $C_D = 0.02$.
5. A level flying airplane weighing 2000 lb at 10,000 ft has a wing area of 250 sq ft and is flying at a velocity of 117 ft/sec. If the wing has an aspect ratio of 7.5 and $C_D = 0.07$, find the parasite drag.

Interference. It is advisable to figure the total parasite drag for the complete model. The reason for this is the existence of a mutual interference between the different parts of the airplane. For instance, when the lift, drag, and moments about the center of gravity are found for, say, the wing and tail surfaces separately, and then the data for the two combined are compared, there is usually a great difference. Actually the lift, drag, and moments of the airplane cannot be figured from flight tests, but a test of a large model in a high-speed wind tunnel is highly satisfactory.

The drag of the separate parts of the airplane may be found accurately, but interference between adjacent parts is impossible to estimate. If two struts join each other at an acute angle, where the wing joins the fuselage or where a number of parts of the airplane are joined together, the air is pocketed to some extent, causing eddy currents. All sharp corners should be avoided because of the turbulence. Any turbulence means an increase in drag.

Fairing. One of the main objectives in practically all types of airplane design is to attain as high a speed as possible. An increase in drag, of course, will reduce performance. Therefore it is necessary to make an airplane as aerodynamically clean as possible by streamlining the various parts. Angles are smoothed out by *fairing* in order to reduce eddies and turbulence; also, surfaces are kept smooth and regular without unnecessary projecting parts. Fairings, such as sheet balsa and aluminum, may cut down the resistance of a fitting or a landing gear as much as $^1/_{16}$ of its original resistance.

Struts. Although the design trend of our modern aircraft has

turned toward more internal bracing, such as the cantilever type monoplane, many light airplanes still exist that employ outside bracings of some sort. A compression member in any part of the structure of the airplane is referred to as a *strut.* Those struts which are exposed to the air, such as the compression members in the interplane wing bracings, the landing gear, or tail-surface bracings, are streamlined so that they will offer as little resistance as possible to the passage through the air.

Whether the structural shape is a tube or a solid, it has been found that the strongest compression member is circular. Examination of the flow around a cylinder and a streamlined strut (Fig. 4-1) shows clearly that the circular tube offers the greatest resistance. The eddying of the air behind the cylinder is the cause for the increase in resistance. This situation is unfortunate, but with careful computation in the stress analysis, *i.e.*, finding the direction and magnitude of

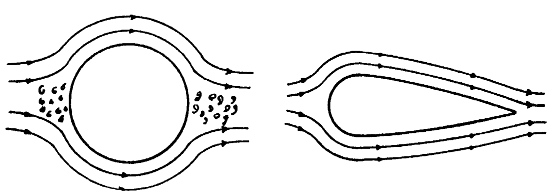


Fig. 4-1. Airflow around cylinder and streamline strut.

the design load, it is possible to use the streamlined strut to great advantage.

A streamlined shape is defined by its *fineness ratio*, which is the ratio of the dimension parallel to the airstream to the dimension across the airstream. The best ratio of structural strength and weight to resistance is of the order of 3 or 4.

A fineness ratio less than 3 shows a big increase in drag coefficient, and a fineness ratio larger than 4 gives a slightly larger drag due, probably, to the added skin friction.

Cylinders and Wires. In modern aircraft design, the use of cylinders and wires has been practically eliminated. The cantilever type wings and empennage have outmoded those extra drag-giving parts. Comparing the cylinder with a strut of equal diameter and a fineness ratio of 3.5, it is found, at the same airspeed, the cylinder gives much greater drag than the strut.

Example: Compare the drag of a 3-in. cylinder with a strut of the same diameter but a fineness ratio of 3.5, if the airspeed is 100 ft/sec.

For the cylinder:

$$R\ (\text{lb/ft}) = 0.00012 \times \text{diameter in in.} \times V^2\ (\text{ft/sec})$$
$$R = 0.00012 \times 3 \times (100)^2$$
$$= 3.6\ \text{lb}$$

For the strut:

$$R\ (\text{lb/ft}) = K \times \text{diameter in in.} \times V^2\ (\text{ft/sec})$$

where K varies with the fineness ratio as follows:

Fineness	K
2.5	0.00000902
3.0	0.00000836
3.5	0.00000812
4.0	0.00000795
4.5	0.00000812

therefore: $R\ (\text{lb/ft}) = 0.00000812 \times 3 \times (100)^2$

$$R = 0.2436\ \text{lb}$$

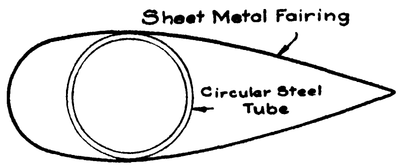


Fig. 4-2. Fairing around cylindrical tubing.

As illustrated in the above example, the cylinder produces a drag of almost 18 times that of the streamline strut. Structural parts must be carefully streamlined.

Formerly, it was the practice to use struts of solid wood, but now seamless metal tubing has taken its place. Also, streamlining has been employed when cylindrical tubing was used. The fairing (streamlining) around the cylinder was not designed as a strength-bearing member but solely as a drag-reducer (see Fig. 4-2).

Generally speaking, two types of round wire are used in airplane construction: (1) solid round wire and (2) stranded wire cable. Resistance of the solid wire can be calculated by the use of the formula for cylinders, as given above. However, when the diameter of the wire drops to 0.2 in., there is introduced an error of about 20% (too high).

As might well be expected, the stranded wire cable, being the rougher in surface, will give greater resistance per foot of length than the smooth wire. Stranded wire gives about 19% greater resistance than the smooth wire of the same size.

For some time it was contended that, although the smooth wire gave less resistance than the stranded cable, the latter was more desirable from the safety viewpoint. The disadvantage of using any hard wire in airplane construction is that, due to vibrations and the reversal of stresses in the structure, the wire may fatigue and snap without any warning. On the other hand, the stranded cable usually gives warning of failure by the cable stretching and the strands failing one at a time. This, of course, makes it a simple matter for the mechanic to find the frayed cable and replace it at once. Nevertheless, the modern methods of fabrication and more careful attention to overall design have eliminated much of the vibration and have reduced to a minimum the likelihood of failure in the smooth wire. Smooth wire is now used exclusively in all external wire bracings and stranded cable is used for bracing where it will not be exposed to the airstream.

PROBLEMS

1. Find the drag of a 6-ft compression strut whose fineness ratio is 4.5 if the diameter is 4 in. and airspeed is 75 mph.
2. A flag pole 30 ft high is constructed of round pipe 4 in. in diameter. What will be the force generated in a 37-mph wind?
3. A turret gun, 3 in. in diameter, is pointed at right angles to the plane of symmetry. If the airplane is flying at 130 ft/sec and the gun barrel is 2.5 ft long, find the drag in pounds.
4. Find the drag of 50 ft of round solid wire, $^5/_8$ in. thick, if the airspeed is 110 mph.
5. Find the drag of two struts of a high wing monoplane if the length of each is 6 ft, with fineness ratio of 2.5 and normal diameter of 2.5 in. at an airspeed of 75 ft/sec.

Fittings. The resistance of fittings at the ends of struts, wires, etc., has been the subject of much speculation and rough approximations. These fittings sometimes extend out into the airstream and add somewhat to the total resistance. It has been suggested by the Navy that a good approximation for the resistance of cable and wire fittings is to "add one foot to length of cable for turnbuckle and one foot for the eye and fitting." For total resistance of strut, use total length, including space occupied by sockets and fittings and add three feet per strut for the additional resistance of the two end fittings." In the cheaper class of airplanes these methods are used, but, where cost is no object, the design of airplanes has terminated the fittings inside the structure and has improved the looks of the airplane and also aided in the "aerodynamic cleanliness."

Fuselages. A difficult and important job in aircraft designing is to estimate correctly the drag coefficient of the fuselage. The only really satisfactory way to obtain the drag of a new design of fuselage

is to test it in the wind tunnel. In order to test a model successfully in the wind tunnel, it must be exact in duplication of windshield, ratio antennas, wires, rivets, etc., and is a very expensive proposition.

If such duplication is impossible, the designer may obtain a very close approximation of the drag of this new fuselage by comparison with one closely resembling his, and which has already had its drag calculated in the tunnel. Knowing the cross-sectional area of the fuselage and the airspeed at which the drag was measured, he may find a coefficient he can use to determine the drag of his own design.

Example: Assume there has been designed an airplane fuselage which resembles the fuselage of another known-type airplane. The fuselage of the older model has a cross-sectional area of 7.6 sq ft, and at 120 mph its drag is 100 lb. The unknown fuselage has a cross-sectional area of 10.5 sq ft. What is its drag at 160 mph?

Solution: From the data of the model already built

$$\text{Coefficient} = \frac{\text{resistance}}{S \times V^2}$$

where S = cross-sectional area of fuselage in sq ft
V = airspeed in mph

therefore

$$\text{Coefficient} = \frac{100}{7.6 \times (120)^2}$$

$$= 0.000914$$

For the newly designed fuselage

$$\begin{aligned} \text{Resistance} &= \text{coefficient} \times S \times V^2 \\ &= 0.000914 \times 10.5 \times (160) \\ &= 246 \text{ lb} \end{aligned}$$

The actual drag on the fuselage increases slightly as the angle of attack increases. This effect is due almost entirely to the body itself; the resistance of the landing gear is considered to remain constant for any angle of attack.

The actual drag curve of the model fuselage, when the square of the speed is plotted against the drag, comes out as a straight line. This shows that the drag varies directly as the square of the velocity. At higher speeds the drag does not vary exactly as the square of the velocity, but is slightly less. However, for all practical purposes the resistance may be considered to vary directly as the square of the velocity.

The resistance coefficients of many modern fuselages range from 0.000220 to 0.000290. By careful design it is possible to cut the resistance coefficients to only a fraction of that for a flat plate.

The N A.C.A. Technical Report 236 gives many additional data on

the drag of fuselages and floats. The drag of a fuselage with rectangular or square cross section is greater than that of the circular or elliptical type. In the attitude of climbing, where drag is of great importance, the rounded section design has proved itself the more desirable.

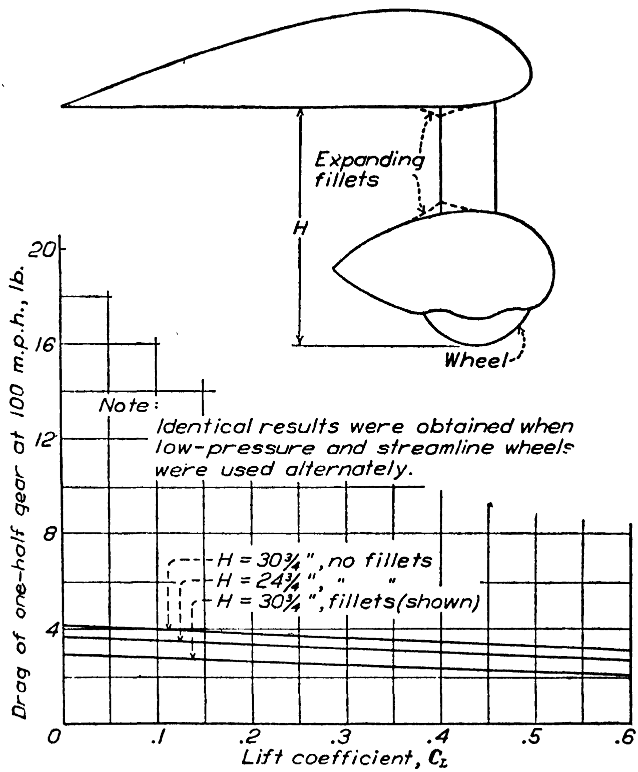


Fig. 4-3. Drag of landing gear B in presence of wing, N.A.C.A.

PROBLEMS

1. An airplane designer does not have available a wind tunnel but desires to find the drag of his newly designed fuselage. He finds that another fuselage, closely resembling his, has previously been tested in a wind tunnel and was found to have a drag of 200 lb at 100 mph. If the known fuselage has a cross-sectional area of 14 sq ft find the drag of the unknown fuselage of 9 sq ft in a cross-sectional area at 90 mph.

2. An airplane fuselage has a cross-sectional area of 8.7 sq ft, at 105 mph its drag is 98 lb. A newly designed fuselage has a cross-sectional area of 14 sq ft and resembles the known fuselage. What is the drag of the unknown fuselage at 140 mph?

3. Find the resistance coefficient of a fuselage with a cross-sectional area of 10 sq ft if the drag at 200 mph is 80 lb.

Landing Gear. Since the early development of the airplane, man has attempted to imitate the flight of birds. One of the more recent advances was the reduction of drag by retraction of the landing gear. It had been known for some time that, if a device could be arranged to pull the landing gear into the body of the airplane, the aerodynamic performance would be greatly improved. Not until one gets

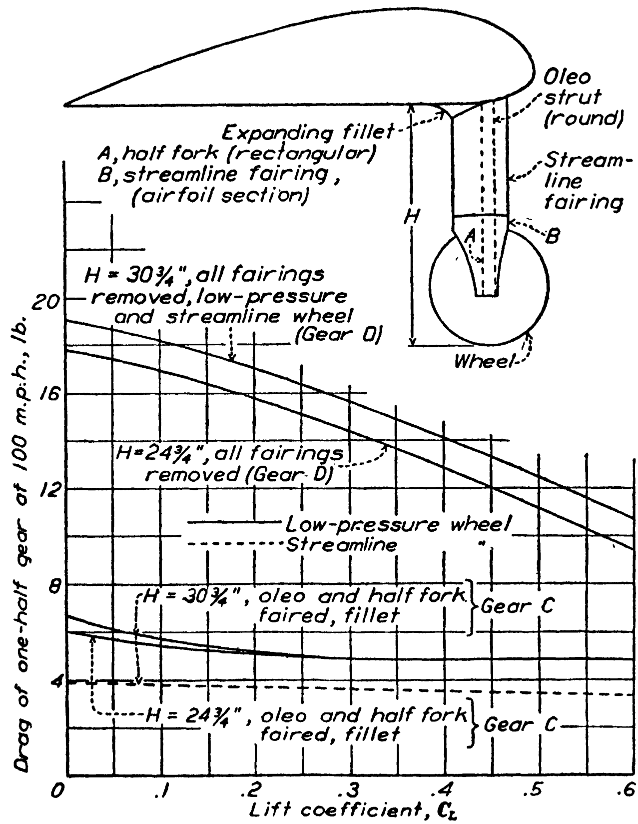


Fig. 4-4. Drag of landing gears C and D in presence of wing, N.A.C.A.

a complete picture of this situation can he fully visualize the problem from a common-sense viewpoint.

There seem to be two channels of thought on the problem of landing gears. First, assume that a designer wishes to build a light airplane for use as a training plane. He does not intend that it be expensive, and high speed is not necessarily the objective. As to landing gear he is confronted with one of two alternatives: (1) install a retractable

landing gear which will add to the cost and will increase the weight of the structure, or (2) design a fixed-type landing gear that will be lighter in weight but will naturally increase the parasite drag. Compare Fig. 4-3 with Fig. 4-4. At different angles of attack, at an airspeed of 100 mph, the differences in drag of the retractable gear and the fixed gear are surprisingly small. Actually there is less than four pounds' difference in the drag of the two gears at 100 mph. It can be seen, by an examination of Figs. 4-3, 4-4, 4-5, 4-6, that, by careful

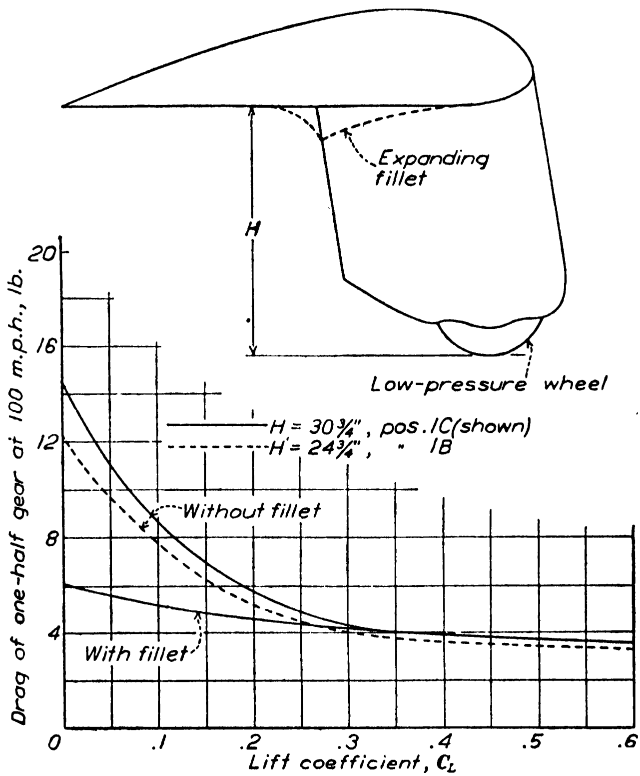


Fig. 4-5. Drag of landing gear A in presence of wing, N.A.C.A.

design and correct use of fairings and fillets, the designer of smaller and lighter airplanes can arrive at an efficient design. The use of the fixed-type landing gear for the light plane class, where the airspeed does not exceed 100 mph, seems to be indicated.

In Fig. 4-4, N.A.C.A. Technical Report No. 518, the fixed-type landing gear, with all the fairings removed, gave a drag of practically 19 lb for one-half gear at 100 mph. The same landing gear fully

faired and equipped with a streamline wheel gave, at the same airspeed, less than 4 lb of drag. It is evident that the necessity for aerodynamic cleanliness cannot be overstressed.

In designing the heavier and faster airplanes the use of retractable landing gears will show a definite advantage. It must be kept in mind constantly that the drag increases as the square of the velocity. In the class of high-speed airplanes, the problem of drag is of utmost

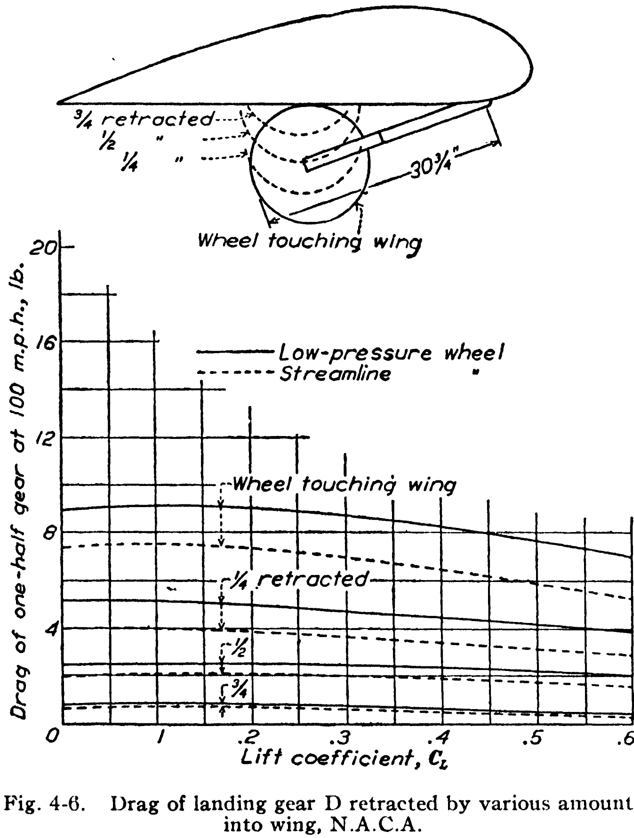


Fig. 4-6. Drag of landing gear D retracted by various amounts into wing, N.A.C.A.

importance. Unlike the light airplane designer, the designer of the high-speed airplane is not chiefly interested in cost of construction; even the added structural weight is somewhat ignored. The additional weight is just another compromise that the designer is eternally making. In this particular instance, however, the reduction in drag is more desirable because high speed is the main objective. An examination of Figs. 4-7, 4-8, 4-9, 4-10 taken from N.A.C.A. Technical Report No. 518 will show a comparison of different types of landing

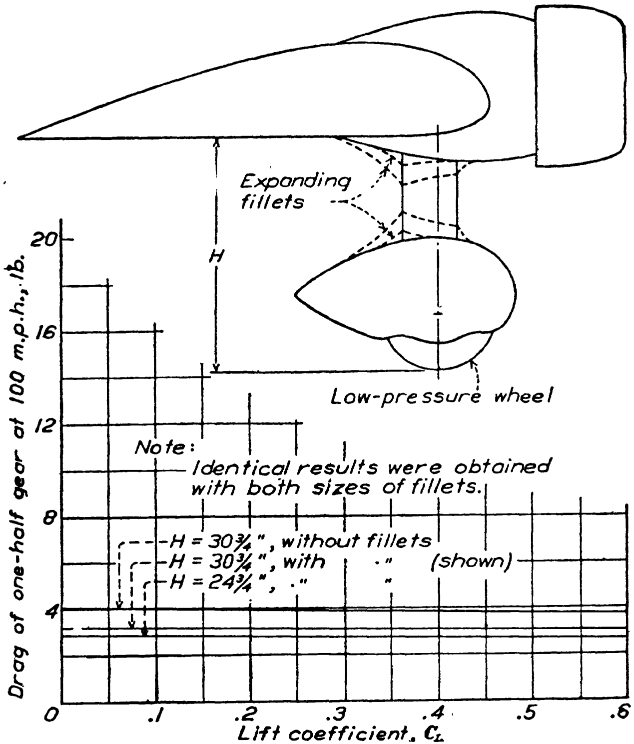


Fig. 4-7. Drag of landing gear B in presence of wing and nacelle, N.A.C.A.

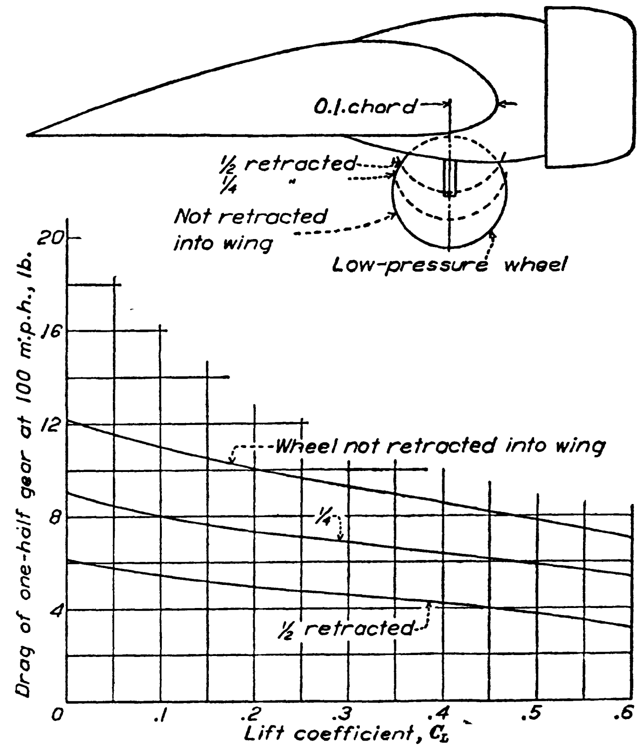


Fig. 4-8. Drag of landing gear D retracted vertically by various amounts into nacelle, N.A.C.A.

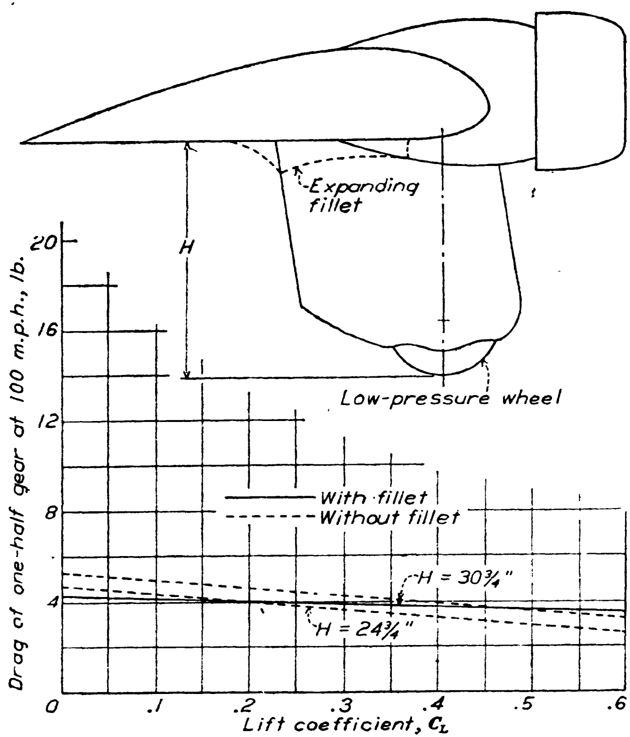


Fig. 4-9. Drag of landing gear A in presence of wing and nacelle, N.A.C.A.

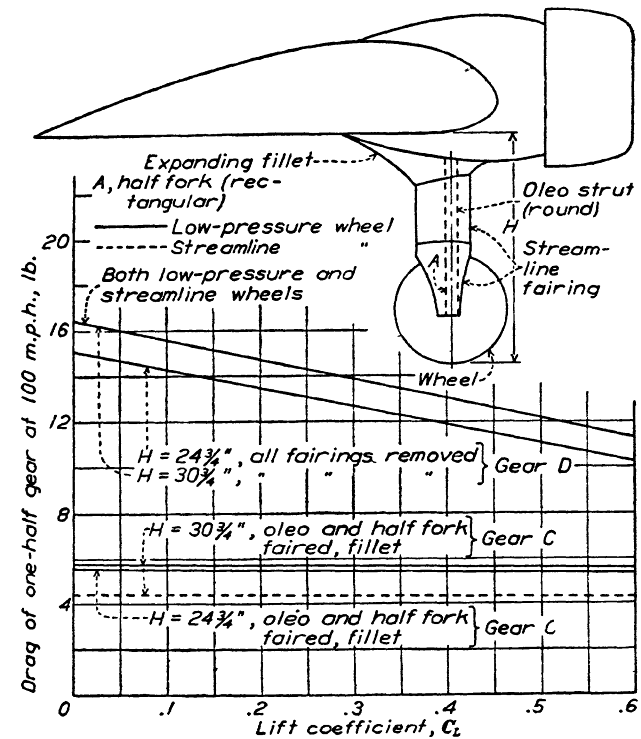


Fig. 4-10. Drag of landing gears C and D in presence of wing and nacelle, N.A.C.A.

gears in the presence of wing and nacelle. Here again, the employment of fairing shows its great importance. The streamline wheel is seen to be superior to the low-pressure wheel. For a full report on landing gears see N.A.C.A. Technical Reports 518 and 522.

Formerly, it has been the custom merely to estimate, roughly, the resistance of individual struts and wheels. However, it has been pointed out, in numerous wind-tunnel tests, that with this type of calculation, the mutual interference of the component parts was entirely neglected. Strangely enough, the interference between the component parts of the landing gear causes a major part of the drag.

If possible, the landing gear should be tested together with the wind-tunnel model of the wing and nacelle, or the fuselage, and the total drag of the combination measured. Figures 4-6 and 4-8 should be of primary interest to the designer of the high-speed airplanes since on the modern types the landing gear is extended at low speeds (take-off and landing). These graphs will enable the designer to calculate the drag at the slower speeds.

In the N.A.C.A. Technical Report 518 on the nonretractable and partly retractable landing gear, the following conclusions were reached:

1. In general, the presence of the engine nacelle did not appreciably affect the drag due to the landing gears.
2. The retractable landing gears were at least one-half retracted into the wing or a fairing before the drag became less than that due to the best nonretractable landing gears.
3. Landing gears partly retracted into a nacelle near the maximum section or partly retracted into the wing near the leading edge had a much higher drag than landing gears partly retracted farther aft on the wing.
4. Streamline wheels used on retractable landing gears had less drag than low-pressure wheels when the landing gear was partly retracted into the wing.
5. Landing gears partly or fully retracted into streamline fairings below the wing had, in general, only slightly greater drag than landing gears partly retracted into the wing or nacelle.
6. The peak or propulsive efficiency was reduced from 1 to 3 percent by the presence of the landing gears tested in conjunction with the propeller.

5

CONTROL SURFACES

Axes. An airplane may rotate about three reference axes, usually centroidal and mutually perpendicular. The *horizontal axis,* in the plane of symmetry, is parallel to and usually is through the propeller axis, and is called the *longitudinal axis.* The longitudinal axis, drawn from rear to front, is also called the X axis. The axis perpendicular to this in the plane of symmetry is called the *normal axis* and is vertical when the airplane is on an even keel. The normal, sometimes called the *vertical, axis,* is referred to as the Z axis. The third axis perpendicular to the other two is called the *lateral axis* or Y axis, and is horizontal when the airplane is on an even keel (Fig. 5-1).

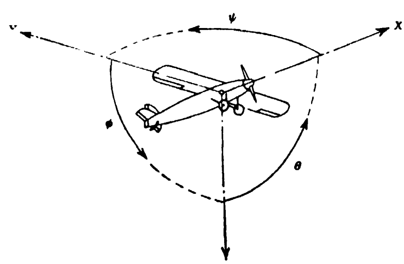


Fig. 5-1. Axes of airplane, N.A.C.A.

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal..	X	X	Rolling....	L	$Y \rightarrow Z$	Roll.....	ϕ	u	p
Lateral.......	Y	Y	Pitching...	M	$Z \rightarrow X$	Pitch...	θ	v	q
Normal.......	Z	Z	Yawing....	N	$X \rightarrow Y$	Yaw....	ψ	w	r

Motions of the Airplane about Axes. An airplane is free to rotate about any one of its axes. The motion about the longitudinal or X axis is termed *roll.* If, when viewed from the cockpit, the airplane rolls in a clockwise direction the roll is positive.

Motion about the lateral or Y axis is called *pitch.* When an airplane dives or climbs, the rotation about the lateral axis is pitch. When an airplane noses down, the pitching moment is a negative or diving

moment. When a airplane is climbing, or is nose-high, the pitching moment is a positive or stalling moment.

Rotation about the vertical or Z axis is called *yaw* or turn. If when viewed from the cockpit the airplane rotates clockwise, the yaw or turn is considered positive.

Vertical Control Surfaces. It is desirable that an airplane be controllable about all three axes. For absolute controllability, an airplane must be equipped with devices to stop rotation about its axes and must fly straight and level, or be able to execute any desired maneuver.

Vertical tail surfaces have been designed so that an airplane may be made to turn at the will of the pilot. The fixed or stationary portion of the vertical control surfaces is the vertical fin and makes up the leading edge of the vertical tail surfaces. The latter portion of the vertical control surfaces is the movable part hinged to the vertical fin and is called the rudder. The *rudder bar,* inside the cockpit, is manipulated by the feet and is connected by cables to the *control horn* which acts as a lever to move the rudder. When the pilot pushes on the right pedal, the rudder is moved to the right and the airplane turns to the right.

Much has been done to find formulas to calculate the vertical tail area necessary for directional stability. In every case, the area necessary for stability has been found to be so small that, in actual practice, the airplane is usually given anywhere from three to four times the required amount. This is because a larger amount of vertical tail area will be required to give adequate directional control on the ground. With conventional-type landing gear, a large rudder is necessary to prevent *ground looping* (see Nomenclature). Designers should eliminate the possibility of ground loops. Along with a large rudder area, the steerable tail wheel and the nose wheel (tricycle landing gear) have been employed, and have been found to improve controllability on the ground.

Fatal accidents resulting from spins caused by improper use of rudder, have led some designers to consider the possibility of designing an airplane without a rudder. Although this seems to be a rather drastic change in design, actually the ailerons can be constructed to give an adequate yawing moment to permit execution of turns. However, one big objection to lack of a rudder is that the airplane cannot be side-slipped; in case of a forced landing this is a serious handicap. Which of two ideas is the more desirable is a design problem.

In designing military aircraft, where great maneuverability is sometimes needed, it is necessary to have a large vertical fin for stability and a rudder effective enough to overcome this stability for maneuvers.

For the preliminary design of commercial airplanes it is customary to have the vertical tail surfaces about 5 percent to 10 percent of the wing area. The rudder area should be anywhere from 40 to 60 percent of the total vertical tail area.

Rudder Travel. In commercial airplane design, the average range of rudder travel is 30 degrees on either side of the neutral position.

Horizontal Tail Surfaces are similar to the vertical tail surfaces in that they are made of fixed and movable sections. The forward portion of the *horizontal tail surface* is usually fixed into the fuselage, and is called the *horizontal stabilizer.* The latter portion, called the *elevator,* is hinged to the stabilizer and is connected by cable and control horn to the control column.

Forward and backward movements of the stick will result in downward and upward movements, respectively, of the elevator. Forward movements of the control column deflect the elevator downward and generate an air force on the lower camber causing the nose to go down (diving moment or *negative pitch*); a backward movement on the stick produces an air load on the upper surface of the elevator and causes a nose-up condition (climbing or *positive pitch*).

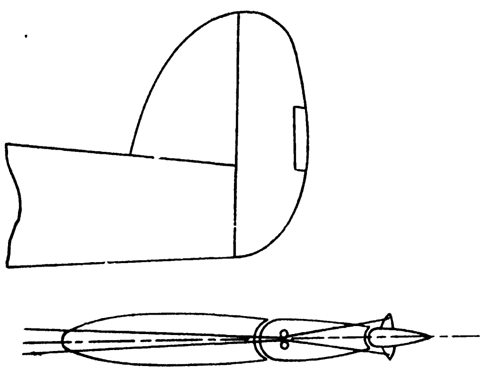


Fig. 5-2. Servo adjustable tab.

The stabilizer and *elevator,* generally speaking, are made of symmetrical airfoils. The degree of movement of the elevators must be limited to a certain amount, because too much freedom of angular range will render them ineffective due to burbling at the higher angles of attack; the airfoil characteristics themselves will determine the allowable amount of travel. As a general practice in commercial designs, the range of elevator travel is from plus or minus 20 degrees to plus or minus 30 degrees. For the design of highly maneuverable military planes, the range may be somewhat greater. Likewise, the

horizontal stabilizer is sometimes made adjustable from the cockpit and the amount of angular travel, both up and down, is limited to only a few degrees. Modern trend in design is to do away with the adjustable stabilizer and use in its place the *servo adjustable tab*. The tab may be either an adjustable or *bent tab* placed on the trailing edge of the elevator for the purpose of trimming the ship for improper flight characteristics, or for aiding the pilot to move a large surface with little effort. The rudder and ailerons may also be equipped with servo adjustable tabs to enable the pilot to use less force in moving a control (Fig. 5-2).

The amount of horizontal-tail-surface area is determined from the length of the fuselage, speed of the airplane, etc., and is discussed to

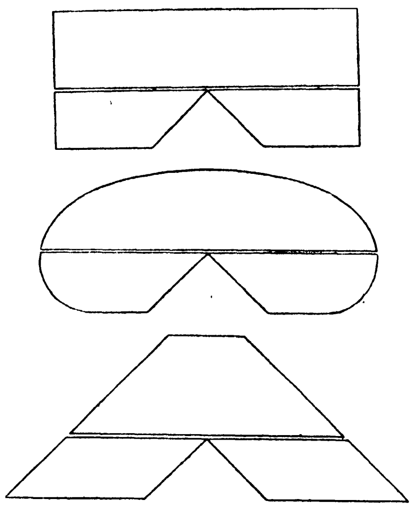


Fig. 5-3. Planform for horizontal tail surfaces.

some extent in the chapter on Stability. Modern design favors a horizontal tail surface that has about 55 percent fixed surface, with the elevator area 45 percent of the total.

There are, in general, three shapes for the planform of the horizontal tail surface: *rectangular*, *elliptical*, and *triangular* (see Fig. 5-3). Although the rectangular planform is more effective aerodynamically, the triangular shape provides for more rigidity in structure. The elliptical planform combines the best features of both rectangular and triangular but is more expensive to build. The final selection, however, must be left to the designer; his decision must be based upon tests on the wind-tunnel model with different sizes and shapes of tail surfaces.

Ailerons. The design of proper *ailerons* has been quite a problem for aerodynamicists because conclusive evidence shows that every airfoil used for a wing requires a different size and shape of aileron to give best results. Not only do size and shape *but also the actual* wing arrangement influence the effectiveness of the aileron.

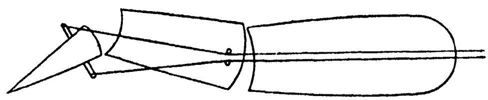


Fig. 5-4. Aileron with cable and horn control.

Conventional design, however, hinges the ailerons so as to make them a part of the trailing edge of the wing. They are moved up and down by cable and control horn attachment to the control stick (Fig. 5-4). A side-to-side movement of the stick produces an up-and-down movement of the ailerons. In flight when the aileron on

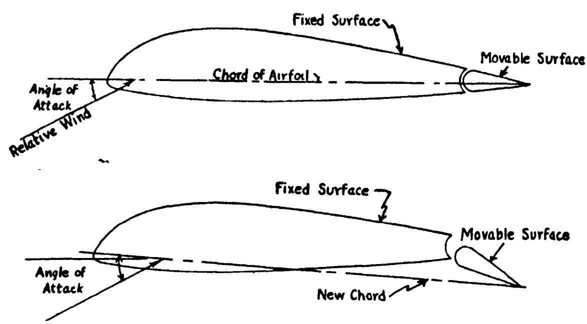


Fig. 5-5. Change of angle of attack with aileron motion.

the right is moved to up-position by moving the stick to the right, the aileron on the left wing simultaneously is moved to down position. That is, when the aileron moves down, the camber is increased and the angle of attack is increased (see Fig. 5-5). The dynamic force of the air strikes the up-turned aileron on the right wing and forces the wing downward. At the same time, the air strikes the lower surface of the down-turned aileron and forces the left wing up. These two actions generate a clockwise rolling moment. When the left wing moves upward for a positive rolling moment, the air force becomes greater on the left wing due to the downward deflection of the

aileron and causes the drag to increase on the left wing and generate a negative yawing moment. When the ailerons are moved for a roll, positive or negative, there is set up simultaneously a yawing moment of the opposite sign. If the yawing moment and rolling moment were of the same sign, over-controlling would result and prove dangerous.

Due to this characteristic action of the ailerons, a *differential* aileron control has been devised that allows the downward movement of the ailerons of only one-half the angular distance that the upward movement travels. For instance, instead of the ailerons moving 15 degrees up and down, the upward movement would be 20 degrees and the downward movement 10 degrees. This arrangement not only gives an improved rolling moment but less yawing moment, due to the decrease in aileron drag on the high wing.

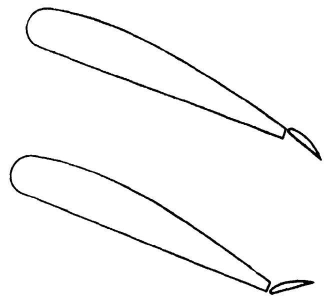


Fig. 5-6. Frise aileron, N.A.C.A.

Another method of reducing yawing moments caused by operating the ailerons is the *Frise aileron*. This type of aileron is mounted for balancing by the use of an inset-hinge and the leading edge of the aileron is shaped (see Fig. 5-6). When the aileron is moved upward, the leading edge of the aileron protrudes below the lower surface of its wing and increases the drag on that side. Thus, the effort of the pilot

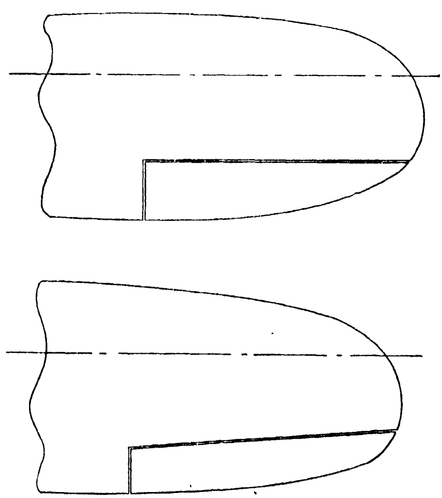


Fig. 5-7. Desirable aileron planforms.

is reduced to a minimum. This action aids also in reducing the adverse yawing moment that would ordinarily be expected.

Design criteria in regard to aileron planform favors the type shown in Fig. 5-7. An investigation made by the N.A.C.A. of effect of airfoil thickness and planform on the effectiveness of the aileron disclosed that the thickness of the airfoil has little effect on the rolling moment or hinge moment. Tapering the airfoil in planform somewhat decreases the rolling moment and hinge moment. However, it was found that the ratio of rolling moment to hinge moment, called the *resulting effectiveness*, was greater for the tapered wing. At the stalling speed, the rectangular planform showed a considerable decrease in rolling moment, whereas, at the maximum lift coefficient, the tapered wing gave greatly improved lateral control.

The correct proportions for ailerons has caused much speculation. A good general rule for aileron area is from 8 to 10 percent of the total wing area if lateral control beyond the stall is not required. The chord of the aileron should be of the order of 20 percent of the wing chord and the span should be about one-half the semi-span of the wing.

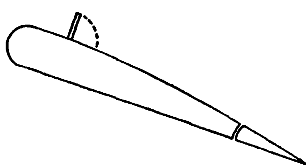


Fig. 5-8. Spoiler.

If lateral control beyond the stall is desired, it will be necessary, perhaps, to use shorter and wider ailerons combined with some sort of *spoiler* (a device to cause a burbling flow over one wing tip in order to reduce its lift, see Fig. 5-8). A combination of both ailerons and spoilers is more effective than either separately.

Balanced Control Surfaces. In addition to balanced control surfaces such as Frise ailerons, there are balanced control surfaces for rudders and elevators. The most popular of these, the *overhang type*, are usually found on the slower light airplanes (see Fig. 5-9). Because a considerable force is required to move a control surface against the dynamic pressure of the wind, control surfaces have been designed so that the axis about which they rotate is back from the front edge. Thus, a portion of the control surface is ahead of the hinge and, as the surface is moved from its original in-line position, the air strikes the front portion and minimizes the effort the pilot must exert. This type of device is not used on faster airplanes because the air load on such a design would tend to bind the hinges and would probably pull off the control surfaces in flight. Tabs on the trailing edge of control surfaces are used in place of the overhand type on the faster military airplanes. With these servo adjustable tabs, the pilot may move a control surface with very little effort.

Propeller Slipstream Action. Although, for most practical purposes, the propeller slipstream is considered to move straight back over the tail surfaces, actually, the propeller imparts a twist to it. The twisting motion, as viewed from the cockpit, is clockwise around the fuselage for propellers that rotate in a clockwise direction. This means that there will be a greater air force exerted on the left side of the vertical tail surfaces. A reaction of this sort tends to make the airplane turn to the left. Owing to the negative yawing moment due to this air force on the vertical tail surfaces, the right wing must travel faster than the left wing, thereby causing more lift on the right wing. This may explain the reason some airplanes have a tendency to fly left wing low, sometimes called *left-wing heaviness.* To counteract

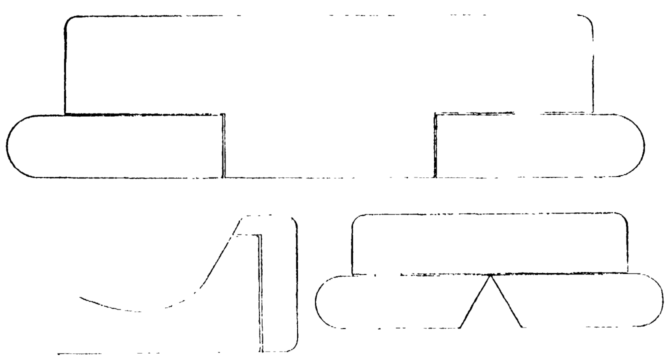


Fig. 5-9. Overhang type of control surfaces.

this, the wings may be twisted in such a way as to increase the *angle of incidence* on the left wing and decrease the angle of incidence on the right wing. When the angle of incidence is increased on a wing it is called *wash-in;* when the angle of incidence is decreased it is termed *wash-out.* Another method of counteracting the *torque effect* of the propeller is to set the vertical fin at a slight angle to the plane of symmetry.

By use of trim tabs, or the above-mentioned methods, an airplane may be made to fly straight and level while cruising but for gliding flight, where the propeller torque action is absent, a slight rudder pressure must be applied in the opposite direction to maintain straight flight.

6

STABILITY

Stability is that property of a body which, when its equilibrium is disturbed, causes it to develop forces or moments tending to restore the original attitude. In the case of an airplane, stability means that, if it is flying at a given angle of attack and that angle of attack is suddenly changed by flying into an up-current, forces will be generated that tend to restore it to the original angle of attack. In this particular case, the airplane would be said to have *longitudinal stability*. If an airplane is flying straight and a force acts on it to switch it off the straight path, forces will be set up which tend to restore the airplane to its original heading. This type of stability is called *directional stability*. In the airplane where no restoring forces are set up, the airplane is said to be *neutrally* stable. When there are forces present

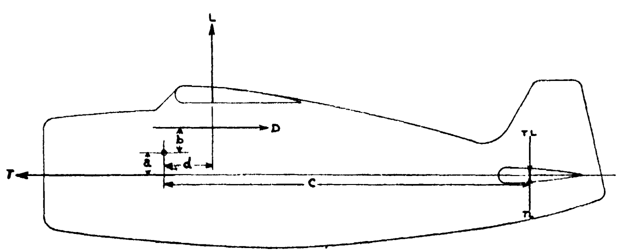


Fig. 6-1. Force system diagram.

which tend to throw the airplane still farther from the original position, an *unstable* condition results.

Forces on Airplane in Flight. The conditions of equilibrium in flight may best be seen by a study of Fig. 6-1. In this diagram the system of forces has been simplified by combining the weights and considering the total to be acting through the center of gravity (C.G.). The C.G. may be computed in the balance diagram of the preliminary design by finding the horizontal and vertical positions of the C.G. from the datum line at the propeller and wheels, respectively. The resultant of drag of the complete airplane is found and at this position the total drag is considered as acting through this *center of resistance.* The lift acting on the wing is considered as the sole lifting force

of the airplane and it, of course, will be acting through the C.P. The lift of the fuselage is very small and actually may be disregarded entirely and the force on the horizontal tail surfaces is considered entirely independent of the total lift, because it may be either an up-load or a down-load. In reality, since all of the quantities of thrust, drag, lift, and their respective moment arms are fixed quantities, *i.e.*, the quantities are fixed by the design of the airplane, the changing of the tail load is the only means for the pilot to maintain equilibrium. In Fig. 6-1, if the airplane is to maintain steady flight, the equations of mechanics (see Introduction) must be satisfied. That is

$$\Sigma H = 0$$
$$\Sigma V = 0$$

and

$$\Sigma M = 0$$

That is to say, the horizontal forces must equal zero:

$$\text{Thrust} = \text{total drag}$$

The vertical forces must equal zero:

$$\text{Lift} = \text{weight} + \text{tail load}$$

Also, the moments must equal zero:

$$(\text{Thrust} \times a) + (\text{drag} \times b) + (\text{tail load} \times c) - (\text{lift} \times d) = 0$$

It can easily be seen that if the moments of lift, thrust, and drag, taken about the C.G. have constant values, in order to have equilibrium in flight, the load on the tail must be maintained according to the attitude of flight. That is to say, the moment arm of the tail is constant in that a symmetrical airfoil is used on the horizontal stabilizer and there will be no C.P. movement with angle of attack. Therefore, the only remaining variable for equilibrium is the load on the tail.

Example: An airplane equipped with a rectangular Göttingen 436 wing with a 42-ft span and 7-ft chord is operating at 120 mph. The E.F.P.A. of the airplane has been computed as 9 sq ft and the C.G. is 18 in. back from the leading edge of the wing. The C.G. is also 10 in. below the center of resistance and 9 in. above the thrust line. The distance from the C.G. of the airplane to the C.P. position of the tail is 22 ft. What must the tail load be for equilibrium if the airplane weighs 2500 lb?

Solution:

$$C_L = \frac{L = W}{\rho/2\ SV^2}$$

$$= \frac{2500}{0.001189 \times 294 \times \left(120 \times \frac{44}{30}\right)^2}$$

$$= 0.231$$

Then from Fig. 3-1, when $C_L = 0.231$, $C_D = 0.013$

$$(\text{Profile drag})\ D_P = C_D\ \rho/2\ SV^2$$

$$\doteq 0.013 \times 0.001189 \times 294 \times \left(120 \times \frac{44}{30}\right)^2$$

$$= 141 \text{ lb}$$

$$(\text{Flat plate drag})\ D_{FP} = 1.28 \times \frac{0.002378}{2} \times A_e \times \left(V \times \frac{44}{30}\right)^2$$

$$= 0.00327 \times A_e \times V^2 \text{ where } V = \text{mph}, A_e = \text{E.F.P.A.}$$

$$= 0.00327 \times 9 \times (120)^2$$

$$= 424 \text{ lb}$$

$$(\text{Total drag})\ D_T = D_P + D_{FP}$$

$$= 141 + 424$$

$$= 565 \text{ lb}$$

then if $\Sigma H = 0$, thrust = drag

$$D_T = 565 \text{ lb} = \text{thrust}$$

From Fig. 3-1, when $C_L = 0.231$, $C.P. = 52$ percent

then

$$C.P. = 0.52 \times 7 \times 12$$

$$= 43.7 \text{ in. back from leading edge}$$

The moment arm of the lift (c) = 43.7 − 18 = 25.7 in.
As determined before:

$$(\text{Thrust} \times a) + (\text{drag} \times b) + (\text{tail load} \times c) - (\text{lift} \times d) = 0$$

Substituting:

$$(565 \times 9) + (565 \times 10) + (\text{tail load} \times 22 \times 12) - (2500 \times 25.7) = 0$$

$$\text{Tail load} = 203.4 \text{ lb}$$

Tail moment is positive (down force).

PROBLEMS

1. With the aid of Fig. 6-1, find the tail load necessary for balance if an airplane is equipped with a rectangular Göttingen 398 wing 60-ft span, 10-ft chord if the airspeed is 85 mph. The C.G. is 17 in. back from the leading edge of the wing, 12 in. below the center of resistance, 7 in. above the thrust line. The C.G. is 20 ft from the C.P. position of the horizontal stabilizer and the E.F.P.A. of the 3850-lb airplane is 3.5 sq ft.

2. An airplane is so designed that, at an airspeed of 100 mph, the load on the tail is 160 lb. The airplane is equipped with a Clark Y (Fig. 3-2) rectangular wing with 48-ft span and 8-ft chord. The C.G. is 10 in. below the center of resistance, 2 in. above the thrust line and 28 in. back from the leading edge of the wing. If the E.F.P.A. has been found to be 2.8 sq ft, find the required distance from the C.G. position to the C.P. position of the tail for balance in level flight if the angle of attack is 3°.

Stabilizer Setting. In the first tests in the wind tunnel, the stabilizer and elevator are first considered as a unit. The unit is constructed so that its angle with reference to the longitudinal axis can be changed through a range of about 6°.

It may be recognized at once, that, with a fixed angle of the stabilizer and elevator, there is only one angle of attack at which equilibrium can be maintained. It is desirable to have this occur at the angle of attack at which the airplane will operate most of the time. If this condition were not obtained the pilot would constantly have to exert a pressure on the control stick in order to fly straight and level.

The varying loads an airplane carries in flight will change the C.G. position, and, to counteract this, on many of the light airplanes an adjustable stabilizer has been installed. The pilot is able to adjust the angle of the stabilizer from the pilot's seat during flight or on the ground. This arrangement is absolutely essential on practically all types of airplanes, in order to relieve the pilot of unnecessary fatigue in keeping constant pressure on the stick. On the larger airplanes, the elevators are equipped with servo adjustable tabs which allow the pilot to trim the airplane in flight. Thus very little effort is required to maintain the desired flight position.

In wind-tunnel tests it has been found that, when the moment curves for the complete airplane are plotted at different elevator settings, the amount the pilot can alter the tail load can be calculated accurately. The airplane will balance, that is, the resulting moments will equal zero, with each elevator setting at a different angle of attack.

Power-on and Power-off. When the model of the complete airplane has been tested in the wind tunnel, the resulting moment curves will be for an airplane without *thrust*. The difference is not to be neglected between this result and one for the same airplane when the engine is running and the propeller is producing thrust. One difference will be the thrust moment about the C.G., and the other is that the air directed back by the propeller has a higher velocity than that of the airplane. Actually, the tail surfaces lie behind the *slipstream* of the propeller and therefore the forces on the tail surfaces will be increased with power-on. The effect of the slipstream and the thrust moment

is to make the airplane less nose-heavy. The *thrust moment* tends to require less of an angle of the elevator at any airspeed with power-on than with power-off.

Thrust Line. The thrust moment and its relation to the balancing characteristics with power-on are seen in Fig. 6-1. Regardless of the position of the thrust line with respect to the C.G., within reasonable limits, the slipstream effect on the empennage will remain approximately the same. Of course, the greater the vertical distance between the thrust line and the C.G. the more drastic will be the difference in the tail loads in power-off and power-on. Placing the thrust line above the C.G. will tend to counteract the tail-heavy effect that the slipstream will impose upon it. This is one of the reasons for using the inverted engine. Due to some other disadvantages, however, it will be sufficient to have the thrust line as close as possible to the C.G., and counterbalance the power-off effect by use of the servo adjustable tab or the adjustable stabilizer.

Position of Drag Moment. In the same manner as the thrust moment, the *drag moment* will increase the tail load as the speed increases, and it is usually located above the C.G. If the speed increases, the drag will increase, and this means a change in the drag moment and a change in the tail moment will have to result to bring about equilibrium. If the *center of resistance* should be placed below the C.G., the condition would be opposite. It follows that the center of resistance should come as close to the C.G. as possible. The conditions of balance for an airplane require that it be balanced for the position and speed of flight for which it is most used. The more the increase of speed beyond that for cruising, the more *tail-heavy* it should become and as the speed is reduced to the minimum the more nose-heavy it should be. This condition, of course, should not be extreme because the increased load on the control column may approach a point where it will require too much strain on the pilot.

Static Longitudinal Stability. An airplane is designed so that, at one desired angle of attack, the resultant of all the forces will be equal to zero. The airplane is then said to "trim" at that particular angle of attack. The requirements for static longitudinal stability can be defined as follows: If, in steady flight at the "trim" angle, the nose of the airplane rises for some reason or other, and the angle of attack becomes greater than the angle of trim, a restoring or diving moment is created (independent of action by the pilot) tending to return the airplane to its original angle of attack; or if, on the other hand, when the nose of the airplane falls for some reason or other, a restoring or stalling moment is created (again without intervention by the pilot) tending to raise the nose and bring back the airplane to

its original angle of attack. An airplane having the characteristics just described is said to have *static longitudial stability.*

If, however, the airplane when displaced from its position of trim to another angle of attack remains at the displaced angle (unless the pilot intervenes) without returning to the original position, it is said to have *neutral static stability.*

Again, if when the airplane is displaced from its angle of trim, the angle of displacement increases indefinitely unless the pilot intervenes, the airplane is said to have *static longitudinal instability* and is potentially dangerous.

For an airplane that has positive longitudinal stability, the plot of the moments about the C.G. against angle of attack should produce a curve with a constant negative slope. Also, when the moments about the C.G. are plotted against air speed, there should result, for longitudinal stability, a straight line with a negative slope. Later, the *static stability* around the vertical axis and the lateral axis will be referred to as *directional* and *lateral stability*, respectively.

Effect of Center-of-Gravity Positions. The most powerful influence on the *inherent stability* of an airplane is its location of the C.G. position with respect to the C.P. position. The most rearward position of C.G. must never move farther back than the most forward position of C.P. It has been proposed that a knowledge of the aerodynamic center and of the position of the C.G. is sufficient for obtaining a criterion for the static stability of the airplane and for computing its magnitude. The *aerodynamic center* of the complete airplane is computed by taking into consideration the arrangement of the wing or wings and tail surfaces only, not upon the angle of attack or their wing sections. The aerodynamic center of an airfoil, referred to earlier, is modified by the computation of the tail surfaces and therefore for the complete airplane will be moved back from the original 25 percent position to approximately the 35 percent position of the chord. When the aerodynamic center coincides with the C.G. the airplane is considered statically indifferent. If the airplane were displaced to some position, by air currents, there would be no tendency for it to restore itself nor any tendency to get worse. This condition, where the resultant air forces pass through the C.G. and produce no moments whatsoever, is termed *neutral* stability. Design and flight tests have proved that it is more desirable for the airplane to possess a limited amount of positive stability: The aerodynamic center of the airplane should be located behind the C.G. If the aerodynamic center were located ahead of the C.G., there would be a moment to increase the angle of attack more and more, creating a tail-heavy condition which is disastrous. If the aerodynamic center

is acting back of the C.G., there will always be produced a negative or restoring moment.

The N.A.C.A. Technical Report No. 672, regarding the effects of C.G. location on a low-wing monoplane, explains that, in nearly every case, moving the C.G. forward steepens the spin and improves recovery. Moving the C.G. back flattens the spin and retards recovery. In a normal spin the airplane wing operates on a high angle of attack. Even though the path of the airplane is vertically downward, which produces relative wind vertically upward, the angle of attack is slightly beyond the burble point. In a *flat spin* the angle of attack is approximately 70° which means that the relative wind is not moving over the airfoil in a way to render it effective. Most tail-

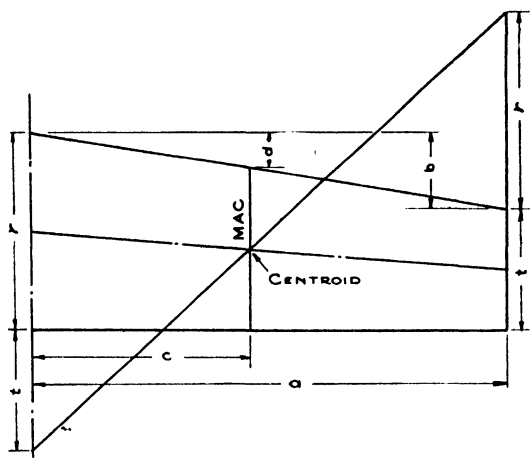


Fig. 6-2. Graphical method to determine M.A.C. of tapered wing.

heavy airplanes will produce a flat spin and recovery is practically impossible. The stability of an airplane that possesses these characteristics has been termed *catastrophic instability*.

Mean Aerodynamic Chord. The chord of the airfoil heretofore referred to has been the chord of a rectangular wing. Likewise, the C.P. position has been given in terms of the rectangular wing chord. However, it may be recognized at once that if any deviation from a rectangular planform is used for a wing, there is no evident wing chord for reference, *i.e.*, if a tapered wing is used, the chord lengths will vary from the root section to the tip.

Since, in stability calculations, the position of the C.P. is of utmost importance, it follows that the length and position of the *average* chord is imperative. In other words, the resultant air force will be

working through a certain point on the *mean aerodynamic chord* (M.A.C.) and the position of this C.P. must be given in percent of this mean chord. The M.A.C. of a wing cellule has been defined as the chord of an imaginary airfoil which, throughout the flight range, will have the same force vectors as those of the actual wing cellule. The imaginary mean aerodynamic chord need not resemble any known airfoil. The wind-tunnel data may be used to calculate the exact length and position of the M.A.C. The chord may also be calculated from the dimensions of the proposed or actual wing; it lies in the plane of symmetry and is parallel to the longitudinal axis of the airplane. The M.A.C. may be obtained from wind-tunnel data but when tests have not been made the following methods are used.

An airplane with a rectangular monoplane wing has its M.A.C. the same as the chord of the wing section.

For a tapered monoplane wing, the M.A.C. of each wing panel is located at the centroid of the plan view of the wing section. Figure 6-2 shows a graphical method of finding the M.A.C. of a wing. The leading edge of the M.A.C. is the line connecting the leading edge of the root and tip sections, and the trailing edge of the M.A.C. is the line connecting the trailing edge of the root and tip sections. This condition denotes a tapered wing. To find the perpendicular distance C from the root chord to the M.A.C.:

$$C = \frac{a(r + 2t)}{3(r + t)}$$

where C = perpendicular distance to M.A.C. from root chord
a = perpendicular distance from root chord to tip chord
r = length of root chord
t = length of tip chord

To find the length of the M.A.C.:

$$\text{Length of M.A.C.} = \frac{2}{3}\left[r + t - \frac{rt}{r + t}\right]$$

To find the distance, d, in a swept-back wing, from the leading edge of the M.A.C. to the leading edge of the root chord:

$$d = \frac{b(r + 2t)}{3(r + t)}$$

When a monoplane has a straight center section and swept-back wing, the M.A.C. of the center section and the outer section are found separately (Fig. 6-3). Then the distance, l, from the center line of the airplane to the M.A.C. is:

$$l = \frac{eS_c + fS_t}{S_c + S_t}$$

where e = distance from center line of airplane to M.A.C. of half the center section

S_c = total area of center section

= distance from center line of airplane to M.A.C. of outer section

S_t = total area of both wing sections

To find the length of the M.A.C. of wing with straight center section and swept-back outer section:

$$\text{Length M.A.C.} = C_t + \frac{(C_c - C_t)(f - l)}{(f - e)}$$

where C_t = length of M.A.C. of outer wing section

C_c = length of M.A.C. of center wing section

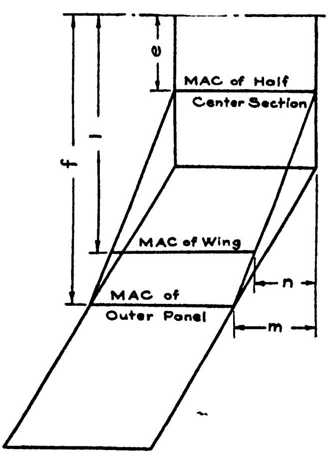


Fig. 6-3. Graphical method to determine M.A.C. of straight center section and sweepback outer panel.

The distance, n, of the leading edge of the M.A.C. back of the leading edge of the center section is

$$n = \frac{m(l - e)}{(f - e)}$$

If the student desires more complete information, almost any good aviation handbook will give him the standard procedure for finding the M.A.C. of monoplanes with dihedral or for biplanes, etc.

PROBLEMS

1. A rectangular wing with a 5-ft chord and 40-ft span has its maximum forward position of C.P. at 30 percent of the chord from the leading edge. The C.G. was found to be back of the C.P. position and to adjust this situation, the center section, 4 ft wide, was left unaltered. The outer panels of the wing were left as parallelograms with area unchanged, but the tip was given a sweepback of 2 ft. (a) Find the distance from the center line of the airplane to M.A.C. of the altered wing. (b) Find the distance that the leading edge of M.A.C. has been moved back.

2. Find, in Problem 1, the distance from the center line of the airplane to the M.A.C. of the wing if the sweepback of the tip was only 1 ft.

3. In Problem 1 the center section is unchanged and the trailing edge of the wing is left perpendicular to the center line. The leading edge of the wing, however, is given a 2-ft sweepback at the tip, and the span is lengthened so that the wing area will remain the same. Find the distance from the longitudinal center line of the airplane to the M.A.C. of the altered wing.

4. In Problem 3, find the distance that the leading edge of the M.A.C. has been moved back.

5. A monoplane with a rectangular wing with a 6-ft chord and 60-ft span has its 5-ft center section unchanged but the leading edge is given a 3-ft sweepback at the tip (starting from the end of the center section), but the trailing edge remains perpendicular to the center line. The area of the altered wing is unchanged (360 sq ft.) (a) Find the new span of the wing. (b) Find the distance from the center line of the airplane to the M.A.C. of the altered wing.

6. Find the length of the M.A.C. in Problem 5.

Pitching-moment Curves. The actual pitching-moment curves of the complete airplane from the wind tunnel are a good indication of the conditions of flight without power. The designer may estimate the thrust moment and slipstream effects, while the proposed airplane is still on the drafting board. With these the longitudinal stability is determined. In the wind tunnel the complete model is tested at the useful ranges of the horizontal stabilizer. The pitching moments of the full-scale airplane about the C.G. may then be plotted (as the ordinate) versus angle of attack (as abscissa). It is seen that the point where angle of attack cuts the X axis, the moment is zero, $\Sigma M = 0$; that is, this is the angle of attack for that particular stabilizer setting where flight equilibrium exists. The criterion for satisfactory static longitudinal stability is that, when the moments about the C.G. are plotted against angle of attack, the slope of the curve should be negative throughout. At any point along the curve the slope should be negative throughout; if the slope is positive, the airplane is statically unstable. Where the curve is parallel to the X axis, the airplane is found to be neutrally stable. The steeper the negative slope of the moment curve, the greater will be the restoring moment when the angle of attack is changed. As was mentioned, too great a restoring moment is undesirable for good performance. Figure 6-4a is an example of pitching curves that illustrate excessive positive stability. In recent tests it has been found that placing the C.G. as far as 20 percent of the M.A.C. from the leading edge tends to make the system very stable. The objective of the designer is never to place the C.G. behind the most forward position of C.P. For most airfoils, the most forward position of C.P. usually occurs at 30 percent of the M.A.C. from the leading edge.

In Fig. 6-4b this type of moment about the C.G. is the criterion for *static longitudinal stability*. The pitching moment is stable so that the airplane tends to maintain a constant flight condition. However, the forces generated in this example are small and within the physical capabilities of the pilot. It has been found that the best position for the horizontal location of C.G. is in the range of 25 to 30 percent from the leading edge of the M.A.C.

In Fig. 6-4c is a typical example of an airplane that has the characteristics of instability in the cruising-speed range. This type of instability tends to make the airplane "hunt" and requires the constant effort of the pilot to control it. This tendency of an airplane is called *phugoid oscillations*. The airplane is stable at higher speed (negative angles of attack) and at slow speed (high angles of attack).

In Fig. 6-4d the type of instability has been appropriately called "catastrophic instability." This particular airplane is stable at high

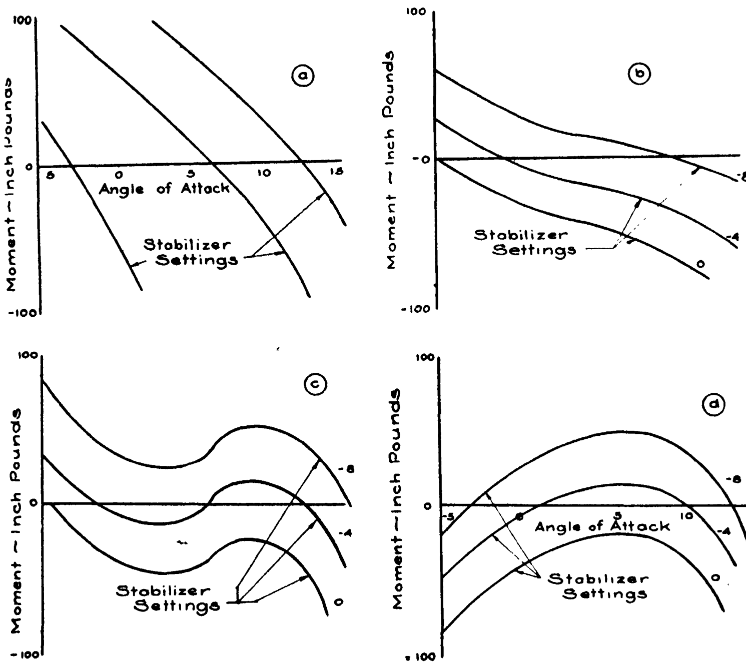


Fig. 6-4. Stability curves for airplane.

angles of attack but is dangerously unstable at high speed or in a diving attitude.

There are other factors that influence static longitudinal stability besides the horizontal location of the C.G. It has been found in wind-tunnel tests that the vertical position of the C.G. is quite important for stability. When the C.G. is located above the M.A.C., the airplane tends to become less stable than when the C. G. is below the chord. Actually the location of the C.G. below the chord does not change the negative slope of the curve at small angles of attack but at higher angles it has a powerful stabilizing effect.

Tail Plane. The *empennage* of the design is another powerful in-

fluence on the longitudinal stability of the complete airplane. *Trim* at any speed requires that the pitching moment due to the tail surfaces be equal to the pitching moments due to the lift and the thrust. If the tail surfaces are located *below* the chord line, the stability will be adversely affected at high speed, and if they are located well *above* the chord line, the adverse effect will be at low speed. Experience has proved, however, that in actual flight tests the best control in the stall favors the lower location of the tail. The reason for the concern for the horizontal tail surfaces is because of the downward velocity of the air off the trailing edge of the wing. In reality, all of the drag moments may be neglected in the computation of stability. The designer may, by making the tail moment large enough to balance the lift moment, use this as the criterion for static balance.

It must be remembered, nevertheless, that with a *finite aspect ratio*, there must be induced drag. This means that regardless of position and angle of attack that the horizontal tail plane makes with the main wing, it will not be the same angle of attack because of the *downwash angle*. Also, as the downwash moves backward toward the tail plane the downwash angle becomes less.

The tail plane is usually located about two-and-a-half to three chord lengths back of the wing. It can be seen at once that, for a given tail, increasing the distance between the C.G. and the C.P. of the tail will make the restoring moment greater and usually increases the stability.

The N.A.C.A. Technical Report Number 293 has explained two elaborate methods for the calculation of the horizontal tail area necessary for a statically stable airplane. The methods are based on considerations affecting the slope of moment curves, and accounts for the effects of wing sections, wing aspect ratio, tail aspect ratio, tail length, downwash, and fore-and-aft location of the C.G. Space does not permit a full explanation of this work.

Increasing the area of the tail plane, taking into consideration the downwash from the wing, increases the moment about the C.G. An increase in the tail plane area will mean added structural weight behind the C.G. and also an increase in drag. An increase of both of these quantities is to be avoided whenever possible.

Increasing the aspect ratio of the horizontal tail plane has the same effect as increasing the area, in that the moment of the tail plane about the C. G. will be increased. The reason for this is that it will increase the slope of the lift curve. Another consideration of increasing the aspect ratio is that, as the span is increased, a greater portion will lie outside the slipstream. This will mean that there will not be quite as great a difference between flight with power-off and power-on.

The three criteria for the selections of an airfoil for the tail surfaces are:

1. The slope of the lift curve should be as great as possible.
2. The airfoil chosen must produce as small a profile drag as possible, at all angles of attack.
3. In the effort of selecting an airfoil with small drag, caution must be observed in order that the airfoil will not be too thin to retain necessary structural strength.

Calculation of Stabilizer Setting. In order to know the tail moments about the C.G., it is necessary to be able to calculate the stabilizer setting. Before becoming too involved it is necessary to recognize the fact that stability cannot be corrected by a change of the stabililzer setting. Variations in the angle of attack of the horizontal stabilizer can be used for securing equilibrium, that is, for adjusting the C.P. correctly at the C.G. for balance at average flying conditions. If, during most of the flying, the stabilizer creates lift in one direction, curving the stabilizer may be recommended. This cambering of the tail surfaces has practically the same effect as changing its angle of attack. For most conventional airplane designs, however, the stabilizer is symmetrical in shape.

The angle of attack of the stabilizer is further limited because an excessive angle of attack may lead to burbling and loss of equilibrium and stability.

Formulas have been derived for the calculation of the downwash angle at the tail surfaces and although some give good results for one type of design they fall short on others. These formulas are useful when no comparative tests are available. A fair value of the downwash angle at the tail, as assumed by the Army, has been taken as "one-half the angle of attack of the wing measured from the angle of zero lift."

Before going further it is best to point out that the angle of attack of the complete airplane is considered as the angle the relative wind makes with the longitudinal axis. The *downwash angle* is the angle between the direction of the relative wind in front of the wing and the direction of the air after it has passed the wing. The angle of attack of the tail surface is called the *tail angle.* It must be remembered also that the tail angle will depend on the stabilizer setting as well as the angle of downwash, and the downwash depends on angle of attack of the wing, and the aspect ratio.

Example: An airplane has a Göttingen 436 airfoil and stabilizer is set at $-3°$ to the chord of the wing. The wing is set at an incidence angle of $+2°$ and the airplane is at an angle of attack of $+4°$. What is the tail angle?

Solution: Angle of attack of the wing = 2° + 4° = 6°

Angle of attack of tail with undisturbed relative wind = 6° − 3° = 3°

The angle of attack for zero lift of Göttingen 436 is −4.5°

The angle of attack of the wing measured from zero lift is 6° + 4.5° = 10.5°

Therefore, by the Army assumption, the downwash angle = 10.5 × 1/2 = −5 1/4°

Then the angle of tail with the downwash = 3° − 5 1/4° = −2 1/4°

From Fig. 6-5 it can be seen that the slope of the lift curve

$$\frac{\Delta C_L}{\Delta \alpha} = \frac{0.15}{2} = 0.075$$

which is approximately the same for most airfoils with an aspect ratio of 6. Then the angle of attack measured from *zero lift* can be expressed

$$\alpha_{L=0} = \frac{C_L}{0.075}$$

Likewise, the angle of downwash at the tail can be assumed as one-half the angle of attack measured from zero lift.

$$\epsilon^\circ = \frac{1}{2} \times \frac{C_L}{0.075} \text{ (aspect ratio 6)}$$

Example: An airplane has a 300-sq ft wing area with aspect ratio of 6. The angle of attack of the wing is +4° and the stabilizer is set at −8° to the wing chord. If the airplane is flying at 120 ft/sec and weighs 3000 lb, what is the tail angle?

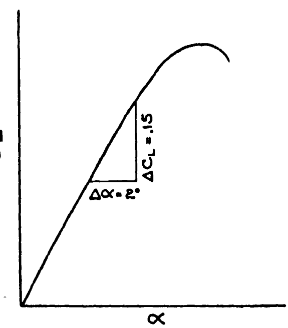


Fig. 6-5. Diagram of slope of lift curve.

Solution:

$$C_L = \frac{L = W}{0.001189 \times S \times V^2}$$

$$= \frac{3000}{0.001189 \times 300 \times (120)^2}$$

$$= 0.584$$

$$\epsilon^\circ = \frac{1}{2} \times \frac{C_L}{0.075}$$

$$= \frac{0.584}{2 \times 0.075}$$

$$= -3.89^\circ = \text{downwash at tail.}$$

Angle of tail with undisturbed relative wind = +4° − 8° = −4°

Angle of tail with downwash = −4° − 3.89° = −7.89°

PROBLEMS

1. An airplane with Göttingen 398 has its stabilizer set at $-5°$ to the chord of the main wing. The airplane is flying at an angle of attack of $+3°$ and the wing is set at $+2°$ angle of incidence. Find the tail angle.

2. A 2000-lb airplane flying 95 mph has a wing area of 250 sq ft (aspect ratio 6). If the angle of attack of the wing is $+6°$ and the stabilizer is set at $-5°$ to the wing chord, find the tail angle.

3. An airplane has a Clark Y airfoil, aspect ratio 6, set at $+2\frac{1}{2}°$ incidence. Angle of attack of airplane is $+7°$. What is the tail angle: (a) when stabilizer is set at $-4°$ to wing chord? (b) when stabilizer is set at $+5°$ to wing chord?

4. A Göttingen 436 wing, aspect ratio 6, is set at $+2°$ angle of incidence to the longitudinal axis. The airplane is flying at $+5°$ angle of attack. Find the tail angle when stabilizer is set at $+5°$ to the wing chord.

5. An airplane, equipped with a Göttingen 398 airfoil is flying at $+3°$ angle of attack. If the angle of incidence is zero degrees, find the tail angle when the stabilizer is set at $+2°$ to the wing.

6. An airplane weighing 5000 lb has a wing area of 370 sq ft, aspect ratio 6, and is flying at 140 mph. Angle of attack of the airplane is $+4°$. The angle of incidence is $1°$ and stabilizer is set at $+6°$. What is the tail angle?

7. An airplane flying at 200 mph at $0°$ angle of attack has a gross weight of 2200 lb and wing area of 230 sq ft. The angle of incidence is $-1°$. Find the tail angle if the stabilizer is set at $+4°$ with the wing chord.

8. Find the tail angle if the stabilizer is set at $-5°$ to wing chord, the angle of attack of the wing is $+3°$ and wing area is 400 sq ft. The airplane is flying at 220 mph and the gross weight is 9000 lb, aspect ratio 6.

9. An airplane has a 240-sq ft wing area with aspect ratio of 6. The angle of attack of the wing is $+5°$ and the stabilizer is set at $-10°$ to the wing chord. If the airplane is flying at 130 ft per sec and weighs 2700 lb, what is the tail angle?

10. An airplane weighs 5200 lb, has 300 sq ft of wing area, aspect ratio 6, and is flying at 160 mph. If the angle of attack of the wings is $+6°$ and stabilizer is set at $+4°$ to wing chord, what is the tail angle?

If it is desired to find the downwash angle at the tail and the main wing does not have an aspect ratio of 6, it will be necessary to correct the angle of attack. Earlier, it was stated that the difference in angle of attack for a difference in aspect ratio was due to the difference in induced angle of attack, that is

$$\alpha - \alpha_1 = \frac{57.3C_L}{\pi AR} - \frac{57.3C_L}{\pi AR_1}$$

then

$$\alpha_1 = \alpha - \frac{18.24C_L}{AR} + \frac{18.24C_L}{AR_1}$$

where α = given angle of attack (degrees)
α_1 = new angle of attack for AR_1
AR = given aspect ratio
AR_1 = new aspect ratio

Finding the angle of attack of zero lift for the airfoil by reference to the lift curve and adding it to the new angle of attack will give the angle of attack measured from zero lift. Then the downwash angle at the tail is determined by using the Army downwash value (one-half the angle of attack measured from zero lift).

Example: In a preceding problem the angle of attack of the Göttingen 436 was 6° for an aspect ratio of 6 and the angle of attack for zero lift is $-4.5°$. If the aspect ratio is changed to 8, find the tail angle.

Solution: Angle of attack of tail with undisturbed relative wind = 3° (as previously found).

Angle of attack of wing measured from zero lift was previously found to be = 10.5°

and

$$\alpha_{L=0} = \frac{C_L}{0.075} \text{ (aspect ratio 6)}$$

then

$$C_L = 10.5 \times 0.075$$
$$= 0.786$$

The new angle of attack for A.R. 8 is

$$\alpha(AR\ 8) = \alpha(AR\ 6) - \frac{18.24C_L}{6} + \frac{18.24C_L}{8}$$
$$= 6° - \frac{18.24 \times 0.786}{6} + \frac{18.24 \times 0.786}{8}$$
$$= 6° - 2.39° + 1.79°$$
$$= 5.4° \text{ angle for } AR\ 8$$

Then for A.R. 8, the angle of attack measured from zero lift is $5.4° + 4.5° = 9.9°$

Therefore by Army value the downwash angle at the tail is $9.9° \times 1/2 = -4.95°$

The angle of the tail with downwash is $3° - 4.9° = -1.9°$

For all aspect ratio corrections, the lift coefficient remains constant and thus from the expression

$$C_L = \frac{\Delta C_L}{\Delta \alpha} \times \alpha_{L=0}$$

it is readily seen that with a change in aspect ratio, the angle of attack measured from zero lift ($\alpha_{L=0}$) will be changed and the slope of

the lift curve ($\Delta C_L / \Delta\alpha$) will also change. For instance, in the above example the slope of the lift curve for A.R. 8 will be

$$\frac{\Delta C_L}{\Delta\alpha} = \frac{0.786}{9.9} = 0.0794$$

That is, as the aspect ratio increases the slope of the lift curve (ratio of C_L/α) will be steeper. Likewise, as the aspect ratio decreases the induced angle of attack will be greater and therefore the slope of the lift curve will be less.

PROBLEMS

1. An airplane equipped with Göttingen 436, aspect ratio 6, has the wing at 5° angle of attack. The stabilizer is set at $-7°$ to the wing chord. Find the tail angle if the aspect ratio is changed to 9.
2. In Problem 1 find the slope of the lift curve for aspect ratio 9.
3. An airplane has its stabilizer set at $-10°$ to the wing chord. With an aspect ratio of 6, the Clark Y airfoil has an angle of attack of 4°. Find the tail angle if the aspect ratio is changed to 12. What is the slope of the lift curve for aspect ratio 12?
4. An airplane equipped with Göttingen 398, aspect ratio 6, has the wing at 7° angle of attack. If the stabilizer is set at $-2°$ to the wing chord, find the tail angle if the aspect ratio is changed to 8.5. What is the slope of the lift curve?
5. An airplane flying at 3° angle of attack has its Clark Y wing, aspect ratio 6, at 2° angle of incidence. If the stabilizer is set at $-9°$ to the wing chord find the tail angle if the aspect ratio is changed to 4.5. What is the slope of the lift curve for aspect ratio 4.5?

Calculation of Tail Moment. Once the angle between the downwash and the chord of the tail has been calculated, the *tail moment* can be determined. The tail moment is the lift force on the tail (down or up) multiplied by the distance of the moment arm which is the distance between the C.G. of the airplane and the C.P. on the tail. The horizontal tail plane is a symmetrical airfoil which means there will be no C.P. movement and since the C.G. is fixed, the moment arm will remain a fixed value. The only variable, then, will be the lift force on the tail which is dependent on the angle of attack of the main wing, the angle of downwash, the aspect ratio of the wing and tail, and the angle of attack of the tail. As a rule, the tail plane will be of a smaller aspect ratio than 6 and this means the angle of attack of the tail must be corrected. A peculiarity of a symmetrical airfoil will be noted in that the zero lift line of a symmetrical airfoil coincides with the chord of the airfoil. That is, the angle for zero lift will be the same as the zero angle of attack for symmetrical airfoils. However, it must be kept in mind that if the aspect ratio is less than 6, the induced angle of attack will be increased when the angle of attack of the tail is changed (positive or negative). Therefore, for any angle of attack other than zero angle of attack,

the induced angle will increase with a decrease of aspect ratio and the slope of the lift curve will decrease.

The lift force on the tail angle is found by use of the familiar expression

$$L_T = C_{L_T}\, \rho/2\, S_T V^2$$

where L_T = lift on tail
C_{L_T} = tail lift coefficient
S_T = area of tail
V = velocity, ft/sec

The lift coefficient of the tail, $C_{L_T} = 0.075 \times \alpha_T$, where $0.075 = \Delta C_L / \Delta\alpha$ = slope of lift curve for A.R. 6 and α_T = tail angle measured from zero lift (which, as explained, is, for symmetrical airfoil, the same as the tail angle of attack). It is evident, then, for an aspect ratio less than 6, the above formula for lift force on the tail must be corrected. Since the only variable is lift coefficient ($C_{L_T} = 0.075 \times \alpha_T$) and the angle of attack of the tail will not be affected by change in aspect ratio, the slope of the lift curve is the only value to be corrected.

Example: Find the tail moment when symmetrical horizontal tail surface is 20 sq ft in area, span is 8 ft, stabilizer is set at $-8°$ to main wing. Main wing is Clark Y, aspect ratio 6 at an angle of attack of 6°, airspeed is 120 mph. Distance from C.G. to C.P. of horizontal tail surface is 16 ft.

Solution: Angle of zero lift for Clark Y $= -5°$

$$\text{Army value for downwash angle at tail, } \epsilon = \frac{11}{2}$$

$$= -5.5°$$

$$\text{tail angle } \alpha_T = -7.5°$$

$$\text{Aspect ratio of tail} = \frac{\text{span}^2}{\text{area}} = \frac{8^2}{20}$$

$$= 3.2$$

$$\text{Lift coefficient of tail} = C_{L_T} = 0.075 \times -7.5 = -0.563$$

where 0.075 and $-7.5°$ are slope and angle of attack for A.R. 6. Correcting α_T for aspect ratio of 3.2,

$$\alpha_T(AR\ 3.2) = \alpha_T(AR\ 6) - \frac{18.24 C_L}{6} + \frac{18.24 C_L}{3.2}$$

$$= -7.5° - \frac{18.24 \times -0.563}{6} + \frac{18.24 \times -0.563}{3.2}$$

$$= -7.5° + 1.714° - 3.210°$$

$$= -8.996° = \text{angle of attack for } AR\ 3.2$$

Then the slope of lift curve for A.R. 3.2 is

$$C_{L_T} = \frac{\Delta C_L}{\Delta \alpha} \times \alpha_T$$

$$-0.563 = \frac{\Delta C_L}{\Delta \alpha} \times -8.996°$$

$$\frac{\Delta C_L}{\Delta \alpha} = 0.0627$$

Then, lift force on tail for A.R. 3.2 is

$$L_T = \frac{\Delta C_L}{\Delta \alpha} \times \alpha_T \times \rho/2\ S_T V^2$$

$$= 0.0627 \times -7.5 \times 0.001189 \times 20 \times \left(120 \times \frac{44}{30}\right)^2$$

$$= -346 \text{ lb}$$

$$\text{tail moment} = 16 \times -346$$

$$= -5540 \text{ ft/lb (down force)}$$

PROBLEMS

1. Find the tail moment when symmetrical horizontal tail surface is 30 sq ft in area, span 9 ft, stabilizer is set at $-10°$ to main wing. Main wing Clark Y, aspect ratio 6, at an angle of attack of 4°, airspeed is 160 ft per sec. Distance from C.G. to C.P. of horizontal tail surface is 18 ft. What is the slope of the lift curve for new aspect ratio?

2. An airplane with Göttingen 436 main wing, aspect ratio 6, is flying 100 ft/sec at 9° angle of attack. The angle of incidence is zero degrees and the symmetrical horizontal tail is 25 sq ft in area, span 8.3 ft and is set at $-8°$ to chord of main wing. The distance from the C.G. to C.P. of horizontal tail is 22 ft. Find: (a) slope of the lift curve of the horizontal tail; (b) lift force on the tail; (c) the tail moment.

3. An airplane is equipped with a Göttingen 398 main wing, aspect ratio 6, and is flying 130 mph at 8° angle of attack. The angle of incidence of main wing is $-2°$. The symmetrical horizontal tail is set at $-5°$ to chord of main wing and has a span of 9.5 ft, area 37 sq ft. The distance from the C.P. of the horizontal tail to the C.G. of the airplane is 14 ft. Find: (a) the slope of the lift curve of the horizontal tail; (b) lift force on the tail; (c) the tail moment.

4. An airplane is flying 90 mph at 7° angle of attack. The main wing, whose angle of attack for zero lift is $-3°$, is set at $-3°$ angle of incidence. The distance from the C.G. of the airplane to the C.P. position of the horizontal tail is 14 ft. The horizontal tail surface is 19 sq ft in area, span 6.5 ft, and is set at $-12°$ to the wing chord. Find: (a) slope of the lift curve for horizontal tail; (b) lift force on the tail; (c) tail moment.

5. An airplane with Göttingen 436 airfoil, aspect ratio 6, is flying

at 164 mph. The wing incidence is $-3°$ and the airplane is flying at $9°$ angle of attack. The distance from the C.P. position of the horizontal tail surface to C.G. is 35 ft. The horizontal tail surface has a span of 12 ft and area of 50 sq ft is set at $-11°$ to the main wing chord. Find: (a) the slope of the lift curve; (b) lift force on the tail; (c) tail moment.

Lateral and Directional Stability. Some authors attempt to consider *lateral* stability and *directional* stability separately. However, the motions of *yaw, roll,* and *sideslip* are so interrelated that the two can hardly be considered separately. Actually, the identical forces that produce motion in roll also produce motion in yaw, although these forces are frequently more effective on the one motion or the other. When an airplane is tipped to one side by up-currents and the inherent characteristics of the airplane restore it to its original position, the airplane has lateral stability. In some cases an airplane, when displaced sideways, returns to its original position and then is displaced to the other side. In other words, this oscillatory motion

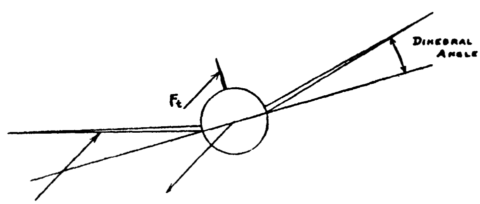


Fig. 6-6. Effect of dihedral in sideslip.

acts in the same manner as oscillations in pitch. For lateral dynamic stability, these oscillatory motions will damp out. To secure restoring moments in lateral motion, *dihedral* is employed in the design. Thus, when sideslip occurs in the rolling motion the lower wing actually has a greater lift produced by a larger angle of attack. The higher wing will have, relatively, a smaller angle of attack and hence there will be a restoring moment (see Fig. 6-6). In the sideslip the *relative wind* will have a side component which, when dihedral is used, will increase the angle of attack on the low wing and decrease the angle of attack on the high wing. Also, if the designer distributes a larger vertical fin above the C.G. than below, the center-of-resistance of the fin area will be located above the C.G. and a restoring moment will be generated when sideslip occurs.

When a large vertical fin area is used far to the rear of the C.G., a yawing moment in the sideslip is produced, *i.e.*, the airplane acts like a weather vane. As the weather-vane effect produced by the vertical tail surfaces heads the airplane into the wind, one wing moves for-

ward faster than the other and therefore, with increased velocity of the wing, a lifting force results. This lifting force tends to increase the bank and the increased bank produces more sideslip, the cycle repeating itself results in a motion known as spiral instability.

Directional stability has been developed by distributing the vertical fin area so that the resultant air forces in a sideslip act to the rear of the C.G. and force the nose of the airplane into the wind. Any excessive directional stability incorporated in the design combines

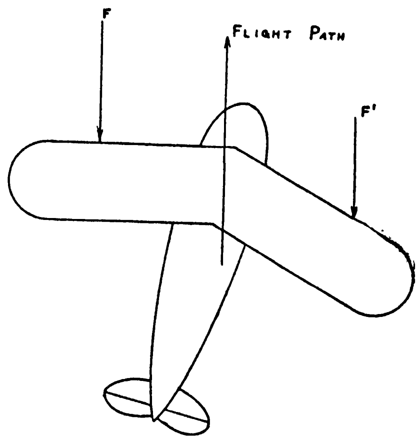


Fig. 6-7. Effect of sweepback in forward slip.

with rolling motion and will produce spiral instability. The designer is constantly faced with the problem of getting the proper amount and position of the vertical fin area to obtain the desired directional stability and still avoid spiral instability.

Sweepback is used in modern aircraft design to obtain the proper horizontal location of the M.A.C. with respect to the C.G. At the same time sweepback has an effect on directional stability (see Fig. 6-7). In a sideslip the drag forces on the two wings are unequal and the result is a stable restoring moment in yaw.

7

LOAD FACTORS

Obviously, to design the structure of an airplane requires knowledge of the loads which will be imposed upon the structure so that the stresses in the members may be determined. With this information the required strength and sizes of the load-bearing members may be calculated. This is a problem for the stress analysts. The purpose of this chapter is to point out the various flight conditions that necessitate great structural strength and to help the student engineer appreciate more fully the responsibility of the designer.

The loads that the airplane are subjected to may be classified as follows:

a. *Air loads* on the main part of the structure, such as the wings and fuselage, and the control surfaces.
b. *Static loads* other than air loads, such as those loads imposed by handling, taxiing, and landing.
c. *Dynamic loads* or any loads which involve the acceleration of the parts or of the complete airplane.

Of these three, the most critical are the dynamic loads encountered in different flight conditions.

After estimating probable loading conditions on an airplane wing, the actual design procedure is not unlike that of a bridge designer. There is, however, a difference in determining the actual *basic load* that must be considered in airplane design. That is, the gross weight of the airplane is considered the basic load and, for unaccelerated level flight, the load that the wing must support is the static weight of the airplane (this is not exactly correct for all flight conditions). Depending upon the maneuver, of course, the load imposed on the wing may be many times the basic load. For accelerated flight this increased load on the wing is called the *imposed dynamic load.*

In any condition of static equilibrium, the basic load is the weight on the structure or part, *i.e.*, it is the load due to the weight of the airplane in unaccelerated level flight. For all types of airplanes in accelerated flight, the structure will be subjected to a load not in excess of the so-called *maximum probable load,* also called *maximum applied load.* Actually, the maximum applied load may be limited by the inherent flight characteristics of the airplane or by a placard or

notice prohibiting various maneuvers that might possibly impose too great a load on the structure. The *applied load factor* is the ratio of the maximum applied load to the basic design load (gross weight).

Even though the problem of the stress analyst is practically the same as that of a bridge designer, the aeronautical engineer has a much more tedious job than the bridge builder. The airplane designer is continually designing and redesigning parts in an effort to reduce to a minimum the structural weight; this, of course, necessitates extremely close design of all parts. The design of a section is made so that the part will sustain an ultimate load (yield load) which is usually fixed at 1.5 times the applied load. The ultimate load divided by the maximum applied load is the *factor of safety*. The ratio of the ultimate load to the basic load is the *design load factor*.

Curvilinear Flight. Constant velocity means constant speed in a straight line. The velocity can actually be changed in two ways: speed or direction. The rate of change of either is called *acceleration*. In level, unaccelerated flight, the force of gravity, called weight, is acting downward toward the center of the earth. This force of gravity is the only downward force acting on the airplane. When the airplane is turned, in other words when the direction of velocity is changed, there is brought about an acceleration. This acceleration is always radially inward toward the center about which the airplane is circling at that instant. This force inward is measured by the mass times the acceleration. In order that the turn be perfect it is necessary that force be equal and opposite to the inward or *centripetal force*. This opposite force is called *centrifugal force*.

$$CF = \frac{W}{g}\frac{V^2}{r}$$

where W = weight in lb
g = acceleration of gravity (32.2 ft/sec^2)
V = velocity of the airplane in ft/sec
r = radius of curvature of the flight path in ft

In a properly executed turn, the lift perpendicular to the wings must equal the resultant of the weight and the centrifugal force. If the bank is not correct, slipping or skidding will result: If the bank is too great there will be a slipping inward and downward; if the angle of bank is not great enough the airplane will skid or move outward.

If the lift is greater than that required for a perfectly coordinated turn the airplane will execute a climbing turn. If the lift is not great enough to counteract the downward resultant force the airplane will settle down in the direction of the resultant force. Figure 7-1 shows

the vector diagram of a plane in correct bank. If angle ϕ is the angular displacement of the wing in the bank, then

$$\tan \phi = \frac{C.F.}{W}$$

$$C.F. = \frac{W}{g}\frac{V^2}{r}$$

where g = acceleration of gravity
r = radius in ft
V = velocity in ft/sec

$$\tan \phi = \frac{\frac{W}{g}\frac{V^2}{r}}{W}$$

$$\tan \phi = \frac{V^2}{gr}$$

It is apparent, then, that the angle of bank is independent of size of the airplane, wing area, etc., provided they have the same airspeed and the same radius of turn.

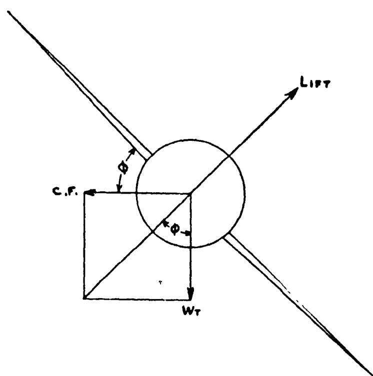


Fig. 7-1. Vector diagram of airplane in bank.

To hold the airplane in any degree of bank the lift on the wing must be equal and opposite to the resultant down force.

$$\text{Lift} = \text{resultant} = \frac{\text{weight}}{\cos \phi}$$

Power required for an airplane in flight varies as the drag and the velocity. Examining the above expression for lift required in a bank, it can be seen that if the angle of bank is greater than zero, the lift on the wing will be greater than the weight because the cosine will be less than one and must be divided into the weight. Lift can be increased for a given airplane in two ways only: either by increasing

the speed or increasing the angle of attack. Increasing the speed or increasing the angle of attack, which necessitates an increase in drag, will require more horsepower in a turn than in level flight.

Example: An airplane with a rectangular Göttingen 436 airfoil has a 36-ft span and a 6-ft chord. It has an E.F.P.A. of 4 sq ft and weighs 4800 lb. It is flying straight and level at 120 mph and is placed in a 40° bank with the same airspeed.

(1) What is the required angle of attack in level flight?
(2) What is the horsepower required in level flight?
(3) What is the centrifugal force in the turn?
(4) What lift is required in the turn?
(5) What is the angle of attack in the turn?
(6) What is the required radius of turn?
(7) How much more HP is required in the turn than for level flight?

Solution:

(1)
$$C_L = \frac{L = W}{0.001189 \times S \times V^2}$$

$$= \frac{4800}{0.001189 \times 216 \times \left(120 \times \frac{44}{30}\right)^2}$$

$$= 0.605$$

From Fig. 3-1 when $C_L = 0.605$

$$\alpha = 3.5°$$

(2) From Fig. 3-1 when $C_L = 0.605$

$$C_D = 0.03$$

$$HP \text{ (wing)} = \frac{C_D\, \rho/2\, SV^3}{550}$$

$$= \frac{0.03 \times 0.001189 \times 216 \times \left(120 \times \frac{44}{30}\right)^2}{550}$$

$$= 76.4\ HP$$

then $$HP \text{ (parasite)} = \frac{1.28 \times \rho/2 \times A_e \times V^3}{550}$$

$$= \frac{1.28 \times 0.001189 \times 4 \times \left(120 \times \frac{44}{30}\right)^3}{550}$$

$$= 60.2\ HP$$

Total HP required $=$ HP (wing) $+$ HP (parasite)

$$HP \text{ (total)} = 136.6\ HP$$

(3) $C.F. = W \tan 40°$

$$= 4800 \times 0.84$$

$$= 4030 \text{ lb}$$

(4) Lift $= \dfrac{W}{\cos 40°}$

$$= \frac{4800}{0.766}$$

$$= 6270 \text{ lb}$$

(5) $C_L = \dfrac{L = W}{\rho/2 \times S \times V^2}$

$$= \frac{6270}{0.001189 \times 216 \times \left(120 \times \frac{44}{30}\right)^2}$$

$$= 0.79$$

From Fig. 3-1 when $C_L = 0.79$

angle $= 6.1°$

and $C_D = 0.046$

(6) Radius of turn $r = \dfrac{V^2}{g \tan 40°}$

$$= \frac{\left(120 \times \frac{44}{30}\right)^2}{32.2 \times 0.84}$$

$$= 1144 \text{ ft}$$

(7) HP (wing) $= \dfrac{C_D\, \rho/2\, SV^3}{550}$

$$= \frac{0.046 \times 0.001189 \times 216 \times \left(120 \times \frac{44}{30}\right)^3}{550}$$

$$= 117\, HP$$

HP (parasite) $= \dfrac{1.28 \times \rho/2 \times A_e \times V^3}{550}$

$$= \frac{1.28 \times 0.001189 \times 4 \times \left(120 \times \frac{44}{30}\right)^3}{550}$$

$$= 60.2\, HP$$

Total $HP = 177.2$

Difference of HP required in bank and level flight $= 177.2 - 136.6 = 40.6\, HP$

PROBLEMS

1. An airplane weighing 3000 lb with an E.F.P.A. of 3.1 sq ft has a rectangular Clark Y wing of 8-ft chord and 48-ft span. The airplane flying level at 100 mph is placed in a 50° bank at the same airspeed. Find: (a) the angle of attack for level flight; (b) the HP required in level flight; (c) the centrifugal force in the turn; (d) lift required in the bank; (e) angle of attack in the bank; (f) HP required in the bank; (g) required radius of turn.

2. An airplane flying level at 85 mph is placed in 60° bank at the same airspeed. The gross weight is 1400 lb, E.F.P.A. is 2.3 sq ft, and rectangular Göttingen 398 wing has 6-ft chord and 36-ft span. Find: (a) the angle of attack for level flight; (b) the HP required in level flight; (c) centrifugal force in the bank; (d) lift required in the bank; (e) angle of attack in the bank; (f) HP required in the bank; (g) required radius of turn.

3. An airplane is executing a 30° banked turn of 2600-ft radius. What is the required airspeed?

4. An airplane weighing 2200 lb is banking at 275 mph. The radius of turn is 300 ft. Find the required angle of bank and the centrifugal force.

5. An airplane is executing a 60° banked turn of 1000-ft radius. What is the required airspeed?

Load Factors. Any flight maneuver that brings about an acceleration on the aircraft structure subjects the wings to forces greater than the weight of the airplane. Newton's second law of motion defines force as mass times acceleration. Mass has been defined as weight of the body in pounds divided by the acceleration of gravity in feet per second per second. When the force on the wings has been found for a certain maneuver, the load factor is found by dividing the static weight of the airplane into this force. To calculate this force it is necessary to know the weight of the airplane, the acceleration of gravity at the point where the weight is measured, and the change of angular velocity in unit time.

$$F = Ma$$

where $$M = W/g$$

therefore $$F = \frac{W}{g} \times a$$

The above expression is for a force acting on an airplane that results in an acceleration, a, in feet per second per second, when W is in pounds, g is the acceleration of gravity (32.2 ft/sec^2), and F is in pounds. In actual practice, the accelerations on an aircraft in flight are not expressed in so many feet per second squared, but are usually referred to in units of g. For example, if the acceleration on an aircraft in a pull-out from a dive were found to be 193.2 ft per sec, the acceleration would be expressed as an acceleration of $6g$. That is,

the acceleration of 6g would mean that the force in pounds exerted on the airplane would be 6 times the gross weight.

The *load factor* of an airplane is the ratio of the force on the wings to the weight of the airplane. Figure 7-2 will show that in a perfect turn, *i.e.*, no slipping or skidding, the lift perpendicular to the wings as stated above is equal to the resultant downward force.

$$L = R$$

The resultant downward force is equal to the weight divided by the cosine of the angle of bank,

$$R = L = \frac{W}{\cos \phi}$$

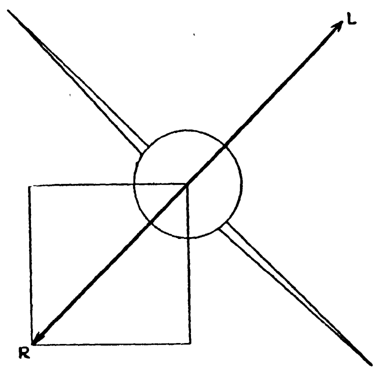


Fig. 7-2. Vector forces of airplane in bank.

The load factor is the ratio of the imposed load to the basic load or gross weight of the airplane.

$$\text{Load factor} = \frac{\frac{W}{\cos \phi}}{W}$$

$$\text{Load factor} = \frac{1}{\cos \phi}$$

It is seen, then, that the load factor in terms of g is dependent only on the angle of bank. In other words, the weight of the airplane, the size, the airfoil, the angle of attack, etc., have no relation to the load factor imposed on the airplane.

Example: What is the load factor of an airplane placed in a 50° banked turn? If the airplane has a gross weight of 3000 lb what is the load imposed on the wings in the bank?

Solution:

$$\text{Load factor} = \frac{1}{\cos 50^\circ}$$

$$= \frac{1}{0.643}$$

$$= 1.56g$$

$$\text{Load imposed on the wings} = 3000 \times 1.56$$

$$= 4670 \text{ lb}$$

Example: An airplane weighing 2000 lb makes a turn of 300-ft radius at 180 mph.

(a) What is the angle of bank?
(b) What is the load factor?
(c) What will be the load imposed on the wing?

Solution:

(a) $$r = \frac{V^2}{g \tan \phi}$$

$$\tan \phi = \frac{V^2}{gr}$$

$$= \frac{\left(180 \times \frac{44}{30}\right)^2}{32.2 \times 300}$$

$$= 7.20$$

$$\text{angle of bank } \phi = 82^\circ\ 10'$$

(b) $$\text{Load factor} = \frac{1}{\cos 82^\circ\ 10'}$$

$$= \frac{1}{0.1363}$$

$$= 7.33g$$

(c) $$\text{Load imposed on wings} = 2000 \times 7.33$$

$$= 14{,}680 \text{ lb}$$

In the above example it can be seen that the slower the velocity of the airplane the smaller will be the radius of turn. In designing faster military ships it is evident that for this higher speed it is necessary to sacrifice some maneuverability.

PROBLEMS

1. An airplane is placed in a 35° bank. What is the load factor and load imposed on the wing if the gross weight is 4000 lb?
2. An airplane weighing 3800 lb makes a turn of 700-ft radius

at 165 mph. (a) What is the angle of bank? (b) What is the load factor? (c) What will be the load imposed on the wing?

3. Find the load factor of a 7000-lb airplane if the angle of bank was 64°. What is the load imposed on the wing?

4. A racing plane rounding the home pylon makes a banked turn of 450-ft radius at 300 mph. If the gross weight is 4000 lb, (a) what is the angle of bank? (b) what is the load factor? (c) what is the load imposed on the wing?

5. An airplane is making a turn of $^1/_2$-mile radius at 200 mph. What is the load factor and load imposed on the wing if the gross weight is 2700 lb?

Stalling Speed in Turns. In the military airplane a small turning radius is desirable. This is accomplished by a reduced velocity or a large angle of bank. Since the load factor $= 1/\cos \phi$ it is evident

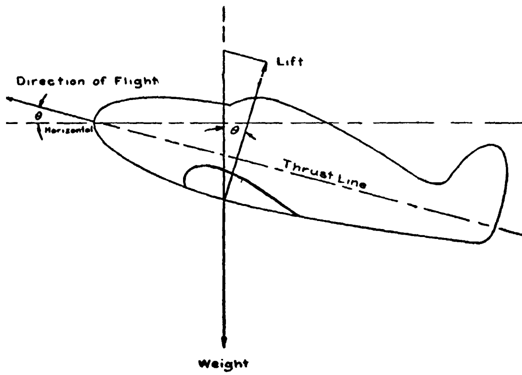


Fig. 7-3. Force diagram of airplane in level flight.

that the larger the bank, the greater will be the stresses on the wing. Also, in the expression for radius of turn

$$r = \frac{V^2}{g \tan \phi}$$

the smallest velocity is related to the angle of bank and it will be shown that the stalling speed of the wing and the power available will be the deciding factor in the performance of the complete airplane.

In unaccelerated rectilinear flight, lift is considered to equal the weight; actually, lift is equal to the weight times the cosine of the angle of attack of the airplane (see Fig. 7-3).

$$L = W \cos \theta$$

A simplification which is almost universal, in this equation, is that $\cos \theta$ is considered equal to one. The maximum angle of climb for

most airplanes is so small that this simplification will lead to no appreciable error in most attitudes of flight. In a bank, however, the lift is equal to the weight times the secant of the angle of bank. The stalling speed in straight flight is

$$V_N = \sqrt{\frac{L = W}{\rho/2\ C_{L\ \max} S}}$$

but the stalling speed in a bank is

$$V_B = \sqrt{\frac{W \sec \phi}{\rho/2\ C_{L\ \max} S}}$$

$$= \sqrt{\frac{W}{\rho/2\ C_{L\ \max} S \cos \phi}}$$

$$= \sqrt{\frac{1}{\cos \phi}} \sqrt{\frac{W}{\rho/2\ C_{L\ \max} S}}$$

$$= \sqrt{\frac{1}{\cos \phi}} \times V_N$$

The above expression means that the stalling speed in a certain degree of bank, V_B, is equal to the normal stalling speed, V_N, times the square root of the load factor for that degree of bank ($\sqrt{1/\cos \phi}$).

Example: An airplane with a Clark Y airfoil and wing area of 200 sq ft and a gross weight of 2500 lb is flying straight and level. What is the normal stalling speed? If at the same airspeed the airplane is placed in a 55° bank what will be the new stalling speed?

Solution: Normal stalling speed in straight and level flight will be

$$V_N = \sqrt{\frac{W}{\rho/2\ C_{L\ \max}\ S}}$$

From Fig. 1-18 $C_{L\ \max} = 1.57$

then
$$V_N = \sqrt{\frac{2500}{0.001189 \times 1.57 \times 200}}$$
$$= 82 \text{ ft/sec} = 56 \text{ mph}$$

Stalling speed in 55° bank $= V_B = \sqrt{\frac{1}{\cos \phi}} \times V_N$

$$= \sqrt{\frac{1}{\cos 55^\circ}} \times 82$$

$$= \sqrt{\frac{1}{0.574}} \times 82$$

$$= 1.34 \times 82$$

$$= 108.2 \text{ ft/sec} = 73.8 \text{ mph}$$

In the above example it can be seen that the greater the angle of bank the higher the stalling speed. In other words, the increased load on the wing in a bank will tend to make the airplane fall out of a bank at a speed well above the normal stalling speed. It is very important that the pilot realize this fact and it is customary to build up the airspeed before going into a turn either by dropping the nose or increasing the throttle setting.

PROBLEMS

1. An airplane has a normal stalling speed of 45 mph. What is the minimum speed in a 65° bank?
2. An airplane equipped with a 300-sq ft wing with a maximum lift coefficient of 1.74 has a gross weight of 2400 lb. Find the normal stalling speed and the stalling speed in a 35° bank.
3. Find the normal stalling speed of an airplane with a wing loading of 18 lb per sq ft and a maximum lift coefficient of 1.42. What is the minimum speed in a 70° bank?
4. An airplane has a wing loading of 20 lb per sq ft and a maximum lift coefficient of 1.67. What is the normal stalling speed at 5000 ft? What is the stalling speed in a 48° bank at that altitude?
5. An airplane stalls out, at sea level, at 35 mph. At what airspeed will the airplane stall out in a 75° bank?

In performance estimation it is interesting to note the difference between horsepower required in level flight and the horsepower required in different angles of bank. In a bank the forces on the airplane increase as the bank increases. With this increase in bank either the velocity must increase or the angle of attack must be increased or both, in order to maintain a perfectly coordinated turn. The horsepower required varies as the cube of the velocity and also increases with increase of drag. In a typical example, Fig. 7-4, the greater the angle of bank the greater velocity and drag and hence the greater the horsepower required. A study of the plot also indicates that a certain limit is placed on the angle of bank. That is, curves may be constructed to show the intersection of horsepower-required curve for a certain degree of bank and the power-available curve. At small angles of bank, where the horsepower-required curves fall below the horsepower-available curve, the minimum speed of the airplane will be the stalling speed for that degree of bank. The range of speeds for these smaller degree of banks will be from the stalling speed up to the intersection of the power-available curves. At this point of intersection will be the maximum speed; beyond that point there is not enough power available to allow higher velocity. For the larger angles of bank, depending, of course, on the design horsepower available, the lower intersection of the power-available and power-required curves determines the minimum speeds. This minimum

speed may or may not be the stalling speed for that angle of bank. As the bank is increased still further, the minimum speed will be at the lower intersection and well above the stalling speed. To determine the stalling speed, the formula

$$V_B = \sqrt{1/\cos\phi} \times V_N$$

may be used. For the larger angles of bank, the top speed will be less on account of the greater power required for the execution of the turn. An inspection of the two intersections of the power-required and power-available curves will reveal that the range of speed will be

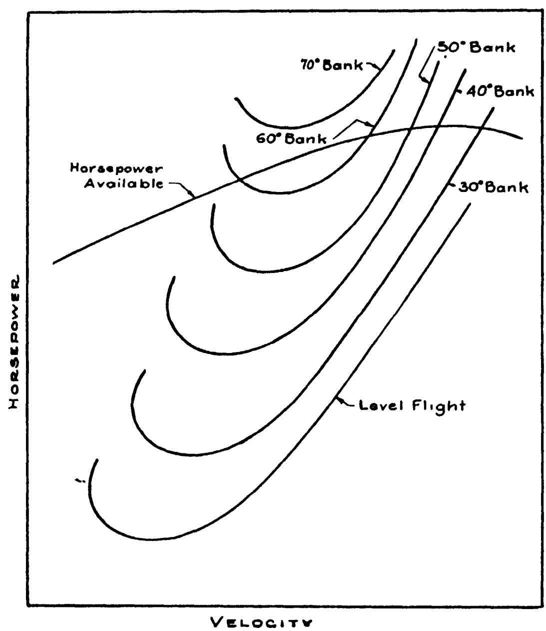


Fig. 7-4. Power required for various degrees of bank.

limited as the bank is increased. For the degree of banks whose horsepower-required curves fall above the power-available curves this indicates that those banks are not possible because there is not sufficient power available.

The above facts are important from the viewpoint of performance. The expression for radius of turn is

$$r = \frac{V^2}{g \tan\phi}$$

For good performance as small a radius of turn as possible is desirable. It can be seen that the smaller the velocity the smaller the radius of

turn, hence the airplane will be more maneuverable. Then, by setting up the curves, horsepower-available and horsepower-required, the lower intersection of these two curves will give the minimum speed for that particular bank. Substituting in the above formula the known values of speed, angle of bank, and acceleration of gravity, the minimum radius of turn may be calculated.

Example: For a particular airplane the HP-required curve for a 30° bank intersects the HP-available curve to give a minimum speed of 54 mph. For the same airplane the HP-required curve for a 60° bank intersects the HP-available curve to give a minimum speed of 75 mph.

(1) Find the minimum radius of turn for the 30° bank.
(2) Find the minimum radius of turn for the 60° bank.

Solution:

(1) Radius of turn
$$r = \frac{V^2}{g \tan \phi} = \frac{\left(54 \times \frac{44}{30}\right)^2}{32.2 \times \tan 30°} = \frac{(79.1)^2}{32.2 \times 0.577}$$
$= 337$ ft radius of turn for 30° bank.

(2) Radius of turn
$$r = \frac{\left(75 \times \frac{44}{30}\right)^2}{32.2 \times \tan 60°} = \frac{(110)^2}{32.2 \times 1.732}$$
$= 217.5$ ft radius of turn for 60° bank.

The radius of turn for the 30° bank is greater even though the velocity was less than that for the 60° bank. The reason for this is because of the greater angle of bank which permits the turn to be made more rapidly. However, the load on the wings for the 60° bank will necessarily be greater than that for the 30° bank because of the greater centrifugal force.

Problems

1. For a particular airplane, the lower intersection of the HP-required and HP-available curves intersect, for a 45° bank, to give a minimum speed of 68 mph. For the same airplane, the available and required curves intersect for a 70° bank to give a minimum speed of 79 mph. Find: (a) minimum radius of turn for 45° bank; (b) minimum radius of turn for 70° bank.

2. In Problem 1, above, if the weight of the airplane is 1500 lb, find (a) the load factor and load imposed on the wing for the 45° bank; (b) load factor and load imposed on the wing for the 70° bank.

Vertical Banks. Pilots have been duly warned to refrain from executing banks in excess of 70° and also from abrupt pull-outs from high-speed dives. No airplane can be made on which the pilot cannot impose extreme loads that will pull off the wings. Many airplanes must be placarded with "Never Exceed" speeds and some must be prohibited from certain maneuvers. It is the wise pilot who knows his own airplane.

An angle of bank in excess of 70° is called a *vertical* bank. Actually a true vertical bank occurs when the angle of bank is 90°. The actual dynamic loads in a vertical bank are difficult to calculate because the airplane settles outward. Regardless of the initial high speed of the airplane, as it enters the vertical bank, there will be such a terrific centrifugal force that the airplane will tend to "squash" or settle outward. This abrupt change in the direction and attitude of flight will increase the drag and therefore decrease the speed. The airplane can be held only momentarily in the position because of the extreme loading and the tendency for the airplane to stall and spin out of the turn if held for any appreciable time.

In the execution of the vertical bank, the only external forces acting on the airplane are the centrifugal force acting outward and the lift on the wings acting inward. This, of course, is assuming no slipping outward and, actually, this condition may exist only for an instant. It is this instant, nevertheless, that interests the designer, because it will represent the most critical loads for the vertical bank. Theoretically then, the lift in a vertical bank should equal the centrifugal force.

$$L = \frac{W}{g}\frac{V^2}{r}$$

and

$$L = C_{L\ max}\ \rho/2\ SV^2$$

then

$$C_{L\ max}\ \rho/2\ SV^2 = \frac{W}{g}\frac{V^2}{r}$$

In the above expression for lift in a vertical bank, the maximum lift coefficient ($C_{L\ max}$) was used because of the fact that the radius of turn will be minimum when the angle of attack is that of maximum lift. It should be noted, however, that even though the $C_{L\ max}$ were used, the corresponding velocity is not the usual stalling speed of the airplane in the bank. The velocity is the initial speed at which the airplane enters the vertical bank and, as discussed earlier, this condition may exist only for a moment before settling outward.

Then if

$$C_{L\ max}\ \rho/2\ SV^2 = C.F. = \frac{W}{g}\frac{V^2}{r}$$

and in straight and level flight,

$$V^2 = \frac{W}{C_L\ \rho/2\ S}$$

Then substituting this value in the above expression

$$C_{L\ max}\ \rho/2\ SV^2 = \frac{W}{g} \times \frac{W}{C_L\ \rho/2\ Sr}$$

Multiplying through by $C_L\ \rho/2\ Sr$ gives

$$C_L\ C_{L\ max}(\rho/2\ S)^2 V^2 r = \frac{W^2}{g}$$

In straight and level flight the lift equals the weight (for practical purposes);

therefore

$$W^2 = (C_L\ \rho/2\ SV^2)^2$$

substituting

$$C_L\ C_{L\ max}(\rho/2\ S)^2 V^2 r = \frac{(C_L\ \rho/2\ SV^2)^2}{g}$$

$$C_{L\ max}\ r = \frac{C_L V^2}{g}$$

$$r = \frac{V^2}{g} \times \frac{C_L}{C_{L\ max}}$$

In the above expression, it is seen that for the absolute minimum radius of turn the lift coefficient will actually be maximum and the velocity will be the normal stalling speed. Therefore, the minimum radius of turn is

$$r_{min} = \frac{V_N{}^2}{g} \quad \text{where } V_N = \text{normal stalling speed}$$

The load factor is the ratio of load imposed on the wings to the weight of the airplane.

$$\frac{L}{W} = \text{load factor}$$

In a vertical bank the load on the wings is

$$L\ (\text{load}) = C_{L\ max}\ \rho/2\ SV^2$$

In the above expression, as explained earlier, the velocity, V, is the initial speed and the value of the lift coefficient will be maximum.

(Momentarily the lift coefficient will be greater than the ordinary $C_{L\,max}$.)

The weight of the airplane in level flight at the normal stalling speed is equal to the lift when the lift coefficient is maximum,

$$W = C_{L\,max}\,\rho/2\,SV_N^2$$

Then, theoretically, the load factor in a vertical bank is

$$\text{Load factor} = \frac{\text{load imposed}}{\text{weight}}$$

$$= \frac{C_{L\,max}\,\rho/2\,SV^2}{C_{L\,max}\,\rho/2\,SV_N^2}$$

$$= \frac{V^2}{V_N^2}$$

where V = initial speed entering bank

V_N = normal stalling speed

The expressions for minimum radius of turn and load factor for a vertical bank contain the normal stalling (V_N). Actually, for a true vertical bank the radius of turn is at its minimum because the angle of bank is maximum and the velocity is the minimum.

Example: An airplane has a normal stalling speed of 55 mph. What will be the load factor imposed on the wings in a vertical bank if the initial speed is 250 mph? What will be the minimum radius of turn?

Solution:

$$\text{Load factor} = \frac{V^2}{V_N^2}$$

$$\frac{(250)^2}{(55)^2} = 20.67g = \text{load factor}$$

$$\text{Minimum radius of turn, } r_{min} = \frac{V_N^2}{g}$$

$$= \frac{\left(55 \times \frac{44}{30}\right)^2}{32.2}$$

$$= 202 \text{ ft}$$

PROBLEMS

1. A racing plane, flying at 300 mph, banks vertically around its home pylon. If the normal stalling speed is 75 mph, what is the load factor on the wings. What will be the minimum radius of turn?
2. The normal stalling of an airplane is 40 mph. What is the ap-

plied load factor in a vertical bank if the initial speed is 150 mph? What is the minimum radius of turn?

3. An airplane with wing loading of 9 lb per sq ft has a wing with a maximum lift coefficient of 1.73. Find the load factor on the wings in a vertical bank if the initial speed is 185 mph. What will be the minimum radius of turn?

4. An airplane has a normal stalling speed of 48 mph. What is the load factor imposed on the wings in a vertical bank if the initial speed is 200 mph? What is the minimum radius of turn?

5. An airplane with wing loading of 20 lb per sq ft has a wing with a maximum lift coefficient of 1.57. Find the load factor on the wings at 10,000 ft in a vertical bank if the initial speed is 140 mph. What is the minimum radius of turn at that altitude?

Acceleration in Pull-outs. It was shown earlier that abrupt changes in the angle of attack brought about a situation where the airplane was operating at the maximum value of the lift coefficient and the original speed instead of the stalling speed. This instantaneous change, while flying at high speed and being brought to the angle of attack for maximum lift, causes the inertia of the airplane to prevent it from changing speed immediately. The result is to bring about an acceleration, that is, V^2/V_N^2.

An airplane in a vertical dive and pull-up experiences the above acceleration which is the same as for that of a vertical bank. Although the expression is the same, an airplane usually goes into a bank from level flight whereas in a dive the velocity may be much greater than the maximum velocity in level flight. Thus, it is evident that the accelerations occurring in a dive could be much greater than those for the vertical bank. The maximum speed obtainable in an airplane is when the airplane is power diving vertically and the drag is equal to the thrust plus the weight. It follows that the maximum acceleration theoretically obtainable would occur when the airplane is instantly pulled out of a power dive at the limiting speed. Mention is made of the theoretical value because, actually, as soon as the pilot begins to pull out of the dive, there is a down-load on the elevator against which he must pull. This change of elevator setting cannot be made instantaneously, and therefore the actual accelerations obtained are smaller than those calculated by the above equation. It is possible to equip the airplane with well balanced elevators that move quickly and easily, making the actual accelerations very nearly the same as the theoretical ones.

When the pilot suddenly pulls back on the stick the tail is thrown downward. Just before this sudden change in angle of attack, the air is flowing smoothly over the airfoil; as the sharp change is made the air instantly breaks away from the upper surface of the wing and causes a separation of the airflow. At this instant the wing is really

acting at an angle of attack that generates a lift coefficient greatly in excess of the ordinary maximum lift coefficient. The load factor here is extremely critical and very dangerous.

In the N.A.C.A. Technical Report No. 203, Lieutenant (now Lt. Gen.) Doolittle made a complete report on the accelerations in various flight maneuvers of the Fokker PW-7, high-speed, pursuit airplane. In this report is included accelerometer records of practically every type of acrobatic maneuver.

The minimum speed of the Fokker PW-7 was approximately 57 mph. Using this value in the equation,

$$\text{Acceleration} = \frac{V^2}{V_N{}^2}$$

the *theoretical* accelerations were calculated and plotted for sudden pull-outs in dives of various speeds. In the PW-7 the control surfaces were balanced almost perfectly and when the actual dives and pull-outs were made the record of the accelerometer was plotted in the same manner as the theoretical calculations. The two calculations, theoretical and actual, were practically identical and it was found that an actual acceleration of 7.8 was found to occur in a sudden pull-out at 162.5 mph.

Example: The PW-7 airplane when actually tested by loading with sand bags was found to be able to hold an ultimate load of 9 times the weight of the airplane. If the stalling speed is 57 mph, what is the "Never Exceed" speed that the airplane should be placarded for?

Solution:

$$\text{Acceleration} = \frac{V^2}{V_N{}^2}$$

$$9 = \frac{V^2}{(57)^2}$$

$$V = 3 \times 57$$

$$\text{"Never Exceed" speed} = 171 \text{ mph}$$

If the PW-7 airplane had been dived at an airspeed in excess of 171 mph and pulled out as quickly as possible, the wings would probably have been pulled off.

Problems

1. If an airplane has been found, by static load test, to withstand a load of 16 times its gross weight, and if the stalling speed is 45 mph, what should be the "Never Exceed" speed?

2. An airplane has been loaded with sand bags and found to withstand a total load of 22 times its gross weight of 1800 lb. The wing loading is 10 lb per sq ft and the maximum lift coefficient is 1.6. Find the "Never Exceed" speed.

Accelerations in Loops. A *loop* is a maneuver so executed that the airplane follows a closed curve approximately in a vertical plane (see Fig. 7-5a). Except for high-speed airplanes, it is usually necessary to dive with power on to gain sufficient speed to carry over the top of the loop. The N.A.C.A. Technical Report No. 203 gives a complete account of five loops made by the PW-7 and the accelerations occurring in these loops. Each loop was made differently either by a change in the initial speed or varied tension on the control column, *i.e.*, the loops were made by pulling the stick rapidly and slowly backward.

In one loop the initial speed was 160 mph and the stick was pulled back quickly, the airplane being allowed to fly itself over. The speed on the top of the loop decreased to 50 mph and in recovery increased to 110 mph. The accelerations experienced in the loops reached 6.1g

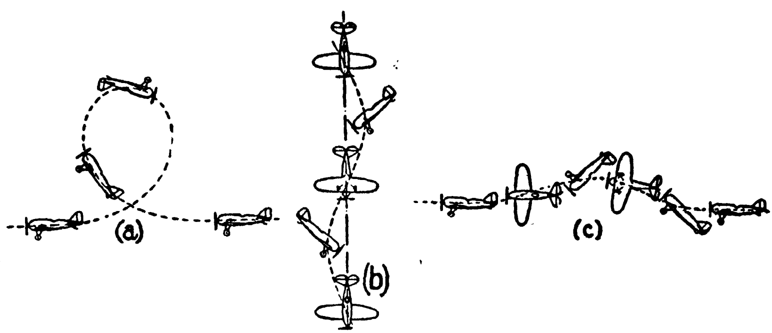


Fig. 7-5. Acrobatic maneuvers, N.A.C.A.

at the initial pull-up and at the top of the loop the acceleration fell to 1g. There was a loss of altitude in this loop.

The slowest loop the pilot was able to make was with an initial speed of 100 mph. The speed at the top of the loop dropped to 30 mph and in pulling out rose to 70 mph. The maneuver was carried out with the stick being pulled back gently but steadily in order to impose as small a load as possible. The maximum acceleration reached in the initial pull-up was 1.5g and decreased to $-0.2g$ at the top. On the recovery a maximum load of 2g was recorded.

This flight test revealed that the dynamic loads which occur in the loop depend upon the abruptness of the pilot when starting the loop. The time required for the different loops made by the PW-7 was from 12 to 18 seconds, depending entirely upon the initial speed of the airplane and the diameter of the loop. Just as in the definition of acceleration (change in velocity per unit of time), the less the time between two given changes in velocity the greater will be the accelera-

tions. In other words, if the pilot dives an airplane at its top speed and pulls out gently and gradually, the accelerations will not be nearly so great as when he pulls up abruptly.

Accelerations in a Spiral. The *spiral* is a maneuver in which the airplane descends in a helix of small pitch and large radius, the angle of attack being within the normal range of flight angles. In the spiral the airplane makes a succession of "loops" in a horizontal plane, in which the airplane gradually loses altitude. The PW-7 airplane made accelerometer records for three spirals with the power on. In the first spiral the pilot attempted to hold the speed and altitude constant. The airplane at first had such a large radius that the acceleration was very small. Then the plane was banked to approximately 70° and pulled into the turn tighter by greater back pressure on the stick. At this moment the acceleration reached $3.3g$ and during the time the airplane was held in the turn with a constant radius, both the speed and acceleration decreased. The airspeed actually dropped from 120 mph to 70 mph and the accelerations varied from $3.3g$ to $1.9g$.

In another spiral the angle of bank was slightly greater than 70° and the radius of turn was made smaller by pulling back more on the stick. This increased angle of attack reduced the airspeed because of the drag increase and this condition required some time for the pilot to establish steady conditions. The speed was held at 140 mph by allowing loss in altitude and, with the power on, an acceleration of $5.5g$ was obtained.

Accelerations in Spins. An airplane in a *spin* is observed to be nose-down, dropping and rotating in a spiral path (see Nomenclature). It has been said that the speed of vertical descent in a normal spin is the stalling speed of the particular airplane. A spin is usually started from horizontal flight attitude by gradually pulling back the stick until the airplane is stalled; at the instant of stall the rudder is kicked, causing the airplane to fall in a sideslip nose down. The airplane will begin to rotate about the longitudinal axis and without further application of the controls the airplane will continue to rotate in the direction in which it was started. This action is called *autorotation.* With the elevator deflected upward it is apparent that the airplane will be working at a high angle of attack. The flight path of the airplane is vertically downward and the relative wind vertically upward (see Fig. 7-5b).

To recover from the spin, it is necessary to reduce the angle of attack; this is done by decreasing the elevator angle, *i.e.*, snapping the stick forward. It is natural for the pilot to try to come out of a spin by pulling back on the stick because the nose is down. If this is

done, however, the airplane will continue autorotation. When the stick is pushed forward the autorotation is stopped but the angle of attack will be decreased and the airplane will continue downward increasing in speed. The airplane will then be operating at a normal angle of attack and it is only necessary for the pilot to pull back on the stick and recover in the same manner as in an ordinary dive. For an airworthiness certificate the Department of Commerce requires an airplane, after a six-turn spin, to recover in no more than one and a half additional turns without use of the engine and with controls in neutral position.

The Fokker PW-7 with the engine throttled was stalled and after turning through about 180° fell into a spin. The accelerations recorded remained practically constant at a value of $2.1g$ for 10 turns. In another spin with power applied the vertical velocity was reported to have been greater than the spin without power. The average accelerations recorded were $2.3g$.

Tail spins have brought about many serious accidents and the causes have varied from poor aerodynamic design to pilot incompetency. Many so-called nonconventional type unspinnable airplanes have been developed by use of combined rudder and aileron controls, *i.e.*, all banks and turns made are perfectly coordinated by the movement of only one control.

The N.A.C.A. has determined some interesting characteristics for normal spins: The radius of the helix was found to be from 1 ft to 10 ft. The velocity in this path was from 60 mph to 70 mph. The number of turns per minute was calculated to be 20 to 40. The amount of altitude lost in each of these turns ranged from 150 ft to 400 ft, depending upon the type of airplane. The angle of attack of the center section of the wing was between 45° and 70°, this of course depending upon the type of spin (see Nomenclature). The attitude of the airplane in the spin was determined for the longitudinal axis and was found to be depressed at the nose 20° to 45° below the horizontal. The inside wing tip dropped 5° to 20° below the horizontal.

Acceleration in Snap Rolls. A *snap roll* or a *slow roll* is a spin when the longitudinal axis is parallel to the horizontal (see Fig. 7-5c). In a snap roll the stick is snapped back quickly and, at the same instant, the rudder is kicked. Some airplanes are easier to snap roll because of the balanced control surfaces.

At the moment a snap roll is begun the acceleration increases to a high value depending upon the initial speed and the abruptness of the pull-back on the stick.

In the PW-7 a quadruple roll started at 160 mph quickly generated a load factor of $7.2g$. A slower and more gradual roll was commenced

at 150 mph. The ship did not whip over as it should and the stick had to be moved forward and then jerked back. When the airplane came out of the spin with a forward speed of 100 mph it was found that the greatest acceleration recorded by the accelerometer was 5.4*g*.

There are many other interesting maneuvers and acceleration records for the Fokker PW-7 and the Boeing PW-9 in the N.A.C.A. Technical Report Nos. 203 and 364.

Inverted Flight. As was pointed out in the chapter on airfoil selection, the type of airplane design will decide the particular airfoil that is best suited for the particular job. Any conventional-type airplane can be flown upside-down but most airfoils are not very effective in this attitude because of the greater upper camber. However, airplanes can be so designed that they will perform exceptionally well in the inverted position. In this particular type of airplane an outside loop can be performed but for the ordinary airplane it should not be attempted. The ordinary airplane is not designed to take the reversed loads imposed upon the wings in this maneuver.

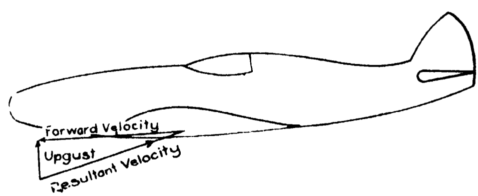


Fig. 7-6. Gust effect on airplane in flight.

Landing Loads. It is not generally believed that the airplane experiences any great dynamic loads in landing. The maximum dynamic loads vary a great deal with the type of landing, that is, for a smooth "three-point" landing the maximum load factor is about 2*g* and the tail-high landing has been found to give a dynamic load in excess of 5*g*.

Gust Loads. Few realize that in rough air, caused by thermal convection, an airplane flying in level flight may be subjected to dynamic loads that exceed those in many violent acrobatic maneuvers. A newly designed airplane, to be deemed airworthy, must pass many exacting inspections and tests for strength and durability. One of the requirements for airworthiness is the ability of the aircraft structure to withstand a sharp-edged vertical gust of 30 ft per sec. By applying hydraulic jacks with special load-indicating devices it is possible to impose upon the underside of the fuselage and wings the ultimate or breaking load that would be generated by the 30-ft-per-sec gust. In this manner the designer may find the ultimate load factor that

the airplane can withstand and then, with a few simple calculations, can determine the "Never Exceed" speed in rough air.

When an airplane, flying level, encounters a vertical up-current of air, the inertia of the airplane tends to keep it moving horizontally forward. From Fig. 7-6 it can be seen that the resultant of the 30-ft-per-sec gust and the horizontal velocity of the airplane produces a relative wind which is directed backward and upward at the airplane. It is not difficult to see that this will result in a high angle of attack and the increase in angle of attack will increase the lift on the wings much more rapidly than the decrease in velocity, due to increased drag. Not only is there a sudden increase in angle of attack but actually the resultant velocity is greater.

Example: An airplane with 200-sq ft Clark Y wing is flying level at 150 mph, at $-2°$ angle of attack. It flies into a sudden up-gust of 30 ft per sec. What will be the new angle of attack, and what is the load factor?

Solution:

From Fig. 7-6 the resultant velocity $V_r = \sqrt{30^2 + 220^2}$

$$= 223 \text{ ft/sec}$$

The angle α, Fig. 7-6, is $\alpha = \tan^{-1} 30/220$

$$= 7.77°$$

New angle of attack $= -2° + 7.77° = +5.77°$

From Fig. 7-6 when angle of attack is $+5.77°$, $C_L = 0.77$

Then new lift $= 0.77 \times 0.001189 \times 200 \times (223)^2$

$$= 9100 \text{ lb}$$

Weight of the airplane before striking up-gust

$$\text{Lift} = \text{weight} = C_L\, \rho/2\, SV^2$$

When angle of attack is $-2°$, $C_L = 0.21$

then

$$W = 0.21 \times 0.001189 \times 200 \times (220)^2$$
$$= 2415 \text{ lb}$$

$$\text{The load factor} = \frac{L}{W} = \frac{9100}{2415}$$
$$= 3.77g$$

A close approximation of the load factor could be quickly calculated by assuming the airspeed unchanged:

$$\frac{C_{La}}{C_{Lb}} = \frac{0.77}{0.21} = 3.67g$$

It is not difficult to see that an airplane flying at higher speed, *i.e*, lower angles of attack, when striking a sudden up-gust will produce a higher load factor than an airplane flying slower and at higher angles of attack.

Example: The same airplane in the above problem is flying at 80 mph and strikes a 30-ft-per-sec up-gust. What will be the load factor?

Solution:

$$C_L = \frac{2415}{0.001189 \times 200 \times \left(8 \times \frac{44}{30}\right)^2}$$

$$= 0.738$$

then $\alpha = 5°$

$$\text{The resultant airspeed} = \sqrt{(117.4)^2 + (30)^2}$$

$$= 121.3 \text{ ft/sec}$$

$$\text{Angle } \alpha = \tan^{-1} 30/117.4$$

$$= 14.3°$$

$$\text{New angle of attack} = +5° + 14.3° = 19.3°$$

then $C_L = 1.55$

$$\text{New lift} = 1.55 \times 0.001189 \times 200 \times (121.3)^2$$

$$= 5430 \text{ lb}$$

$$\text{Load factor} = \frac{L}{W} = \frac{5430}{2415}$$

$$= 2.25g$$

The reason pilots are urged to slow down in rough air is easily seen by the above examples. When the engineer has found the ultimate design load factor and has allowed one-third of the ultimate load factor as the safety margin, he may then state what the placarded top speed of the airplane should be.

PROBLEMS

1. An airplane with Clark Y wing of 180 sq ft is flying at 100 mph, at 0° angle of attack. If the airplane flies into a sudden up-gust of 30 ft per sec, what will be the new angle of attack and the load factor?
2. With the same airplane in Problem 1 (above) find the new angle of attack and load factor if the airspeed is reduced to 90 ft per sec.
3. An airplane with a Clark Y wing of 240 sq ft of area is flying at 200 mph at $-3°$. Find the new angle of attack and the load factor if a 30-ft-per-sec up-gust is encountered.
4. With the same airplane in Problem 3 (above), find the new angle of attack and the load factor if the engine is throttled to 140 mph.

8

PROPELLERS

General Discussion. The main consideration in the design and use of the aircraft propeller is to obtain the maximum performance of the airplane from the horsepower delivered by the engine under such conditions of operation as take-off, climb, cruising, and high speed.

The *propeller* is a device for transforming energy developed by the engine into thrust energy for propelling the aircraft through the air. This propelling force by the propeller is called *thrust.*

The earlier-type propellers were designed primarily in the same manner as a windmill: The thrust force produced was due entirely to the dynamic lift produced by deflecting air rearward from the flat face of the blades. Later, it was discovered that if the blades were considered as wings, or, more specifically, a number of wings of infinitely short span placed end to end, the results would be better. Actually, the treatment of the propeller as a device that produces an aerodynamic reaction is universal and is of primary importance in design.

Since the propeller is connected to the engine it must be able to absorb the power delivered to it by the engine regardless of the speed. Maximum *efficiency* of the propeller at certain speeds is the main design objective. The propeller should be able to transform the rotary power of the engine into forward tractive power. The fixed propeller cannot be efficient at all airspeeds and engine speeds; therefore it is usually so designed as to give its best performance at its design cruising speed. In some cases one outstanding feature such as high top speed, *i.e.*, for a racing plane, may be the object of the design. If a fixed-pitch propeller is used, certain desirable characteristics must be sacrificed. For example, if the fixed-pitch propeller is designed to give exceptionally high top speed, the take-off and climb characteristics will be poor.

Experiment has proved that the larger the diameter of the propeller, the lower the rpm required for the same thrust; and, in general, the larger-diameter propeller is more efficient than smaller propellers because the blade interference is reduced and a higher aspect ratio is obtained. However, there are limitations as to the diameter of the propeller:

a. The greater the diameter, the greater the tip speed.

b. The diameter is limited by the necessity for proper ground clearance.

The maximum tip speed of propeller blades is limited to less than the velocity of sound in air (approximately 1100 ft/sec). At tip speeds above this value, the air *cavitates* behind the blade, rapidly decreasing the efficiency. Geared engines are used that permit high engine speed and at the same time operate propellers at lower speeds, usually about 900 ft/sec. The formula for velocity of sound is

$$\text{Velocity of sound in air} = 65.9\sqrt{T}\ \text{ft/sec}$$

where T = temperature in centigrade absolute

So at an altitude of approximately 40,000 ft (temperature 118° centigrade absolute) the velocity of sound is only 973 ft/sec. It can be seen, then, that an increase of altitude means a great decrease in the velocity of sound and, if the tip speed is the same as the velocity of sound, there will probably be a great loss in power due to violent compression waves that are generated.

PROBLEMS

1. At 14,000 ft the standard temperature is 9.2° F. Find the velocity of sound at that altitude.
2. At 30,000 ft the temperature is −48.1° F. Find the velocity of sound.
3. At 50,000 ft the temperature in the isothermal region is −67° F. Find the velocity of sound.

Momentum Theory. The propeller *momentum* theory, developed by R. E. Froude, and sometimes referred to as *Froude's slipstream theory,* deals with the changes of energy of the mass of air affected by the propeller. Professor Bradley Jones has given the following explanation of the propeller momentum theory.*

"The propeller is assumed to be a disk which exerts a uniform pressure or thrust over the cross section of the air column passing through the disk. It is further assumed that the disk imparts no twisting or rotation to the air column, and that flow remains streamline while coming to the disk, passing through it and beyond it. Referring to Fig. 8-1, X is a point in the air stream far enough in front of the propeller so that the pressure is atmospheric; Y is a point in the air stream far enough to the rear of the propeller so that the pressure is atmospheric.

"Let A = propeller disk area
A_X = cross-sectional area of air column at X

* Reprinted by permission from *Elements of Practical Aerodynamics,* Third Edition, by B. Jones, published by John Wiley and Sons, Inc.

A_Y = cross-sectional area of air column at Y
V = velocity at X
$V(1 + a)$ = velocity at propeller disk
$V(1 + b)$ = velocity at Y
p = atmospheric pressure = static pressure at X and Y
p_1 = static pressure just ahead of propeller disk
p_2 = static pressure just behind disk

"Since the volume of air flowing through each area in unit time is the same,

$$A_X V = A V(1 + a) = A_Y V(1 + b)$$

"No energy is added to or subtracted from the air from X to a point immediately in front of the propeller disk, so Bernoulli's equa-

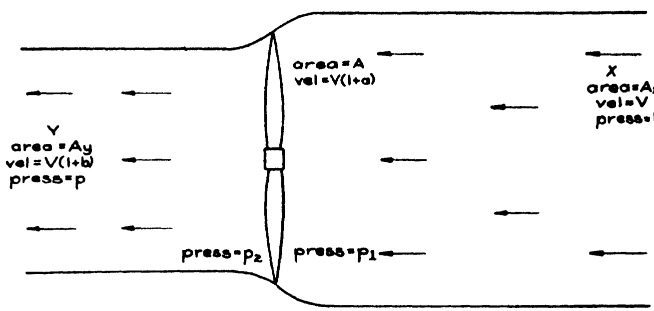


Fig. 8-1. Propeller momentum theory.

tion may be applied, and the sum of static and velocity heads at these two points placed equal

$$p + \rho/2\ V^2 = p_1 + \rho/2\ V^2(1 + a)^2$$
$$p_1 = p + \rho/2\ V^2[1 - (1 + a)^2]$$

"No energy is added to or subtracted from the air from a point immediately in the rear of the disk to point Y, so that the total heads at these points may likewise be placed equal.

$$p_2 + \rho/2\ V^2(1 + a)^2 = p + \rho/2\ V^2(1 + b)^2$$
$$p_2 = p + \rho/2\ V^2[(1 + b)^2 - (1 + a)^2]$$

It will be noted that the velocity immediately in front is assumed to be the same as the velocity immediately in the rear of the disk; otherwise there would be an instantaneous increase in velocity and an infinite acceleration, which cannot be. The propeller disk does exert thrust on the air column, and the thrust (T) is equal to the difference in pressure created by the propeller times the area of the propeller disk.

$$T = (p_2 - p_1)A$$

"Substituting values of p_1 and p_2 already found:

$$T = A\left[p + \rho/2\ V^2([1 + b]^2 - [1 + a]^2) - p - \rho/2\ V^2[1 - (1 + a)^2]\right]$$
$$= \rho/2\ A V^2[(1 + b)^2 - 1]$$
$$= \rho/2\ A V^2(b^2 + 2b)$$

"Thrust is also equal to the rate of change of momentum which is the mass of air affected per second times the change in velocity. The mass of air handled per second is the density multiplied by the volume, and the volume is the cross-sectional area at any point times the velocity at the same point. At the propeller disk the area is A and the velocity $V(1 + a)$ so the mass of air passing through the propeller disk per sec is

$$\rho A V(1 + a)$$

At point X, the velocity is V;
At point Y, the velocity is $V(1 + b)$ or $V + bV$;
therefore the change in velocity imparted to the air column by the propeller is bV.
then

$$T = \rho A V(1 + a)bV$$
$$= \rho A V^2(1 + a)b$$

"Equating this value for T to the one previously found:

$$\rho A V^2(1 + a)b = \rho/2\ A V^2(b^2 + 2b)$$
$$b + ab = {}^1/_2(b^2 + 2b)$$
$$2b + 2ab = b^2 + 2b$$
$$2a = b$$
$$a = b/2$$

"This means that of the total increase in velocity imparted to the air column, one-half is added before the air passes through the propeller disk."

Another conception of Froude's slipstream theory, although not entirely different from the above example, deals with the *work* done per unit time. In the N.A.C.A. Technical Report No. 213, the aircraft is considered moving with a velocity, V, which is considered the free-stream velocity. The propeller sets in motion backward a slipstream, whose final velocity may be called $(V + v)$, the velocity, v, being the total increase in the velocity imparted to the air column. The air actually passing through the propeller has already had imparted to it a portion of this velocity, v, and by a simple application of mechanics, this additional velocity may be proved to be approximately one-half of v. That is, consider the aircraft at rest, as in a wind-tunnel experiment, and placed in a stream of air having the velocity, V. The propeller will be revolving as usual and the velocity of the air passing through

the propeller may be called $(V + w)$. The final velocity of the slipstream, as mentioned above, is called $(V + v)$. Letting m represent the total mass of air per unit time, the thrust, T, of the propeller is,

$$\text{Thrust} = \text{force} = \text{mass} \times \text{acceleration}$$

If the free-stream velocity of the air is V and the total velocity of the slipstream becomes $V + v$, then the acceleration on the air mass in unit time is v.

Then $$\text{Thrust force} = mv, \text{ where } m \text{ is mass}$$

This thrust force, mv, at the propeller disk acts on air moving with a velocity $V + w$. Therefore the work done per unit time is

$$\text{Work} = \text{force} \times \text{distance}$$

Distance is the product of velocity and time, therefore, considering everything in unit time,

$$\text{Work} = mv(V + w)$$

This *work*, $mv(V + w)$, is equal to the increase of the *kinetic energy* of the air, *i.e.*, the work done is the difference between the kinetic energy of the air behind the propeller and the kinetic energy of the air before being worked on by the propeller:

$$\tfrac{1}{2}m(V + v)^2 - \tfrac{1}{2}mV^2 = mv(V + w)$$

$$\frac{mV^2}{2} + \frac{2mVv}{2} + \frac{mv^2}{2} - \frac{mV^2}{2} = mv(V + w)$$

$$mVv + \frac{mv^2}{2} = mv(V + w)$$

$$V + \frac{v}{2} = V + w$$

$$w = v/2$$

Thus, of the total increase of velocity, half of this increase occurred before passing through the propeller. This increase in velocity has been appropriately called the *indraft velocity*.

The efficiency of the propeller which may be called an *ideal efficiency* may be designated by the symbol η_i and is generally given by the equation

$$\eta_i = \frac{\text{output}}{\text{input}} = \frac{TV}{T\left(V + \frac{v}{2}\right)}$$

since TV is the ***propulsive power*** the propeller furnishes, and the pro-

peller contributes $T(V + v/2)$ to the airstream. Simplifying the expression still further by dividing through by TV,

$$\eta_i = \frac{1}{1 + \frac{1}{2}\frac{v}{V}} = \text{ideal efficiency}$$

The ideal efficiency, as expressed above, is never realized since the only loss considered is in the slipstream. The following real losses which are not considered in this theory are:

1. The loss due to interference of the finite number of propeller blades.
2. The loss due to hub interference and tip vortices.
3. The energy loss in the twist of the slipstream by the propeller torque.
4. Lastly, the loss due to the profile drag of the blades.

An examination of the above expression for the ideal efficiency shows:

1. that the ideal efficiency decreases with an increase of thrust,
2. that the ideal efficiency increases with increase of forward velocity,
3. that if the diameter of the propeller is increased the forward velocity will be greater, and hence the efficiency will be increased.

The conclusions are that the diameter of the propeller should be as large as possible and that a propulsive force by means of a small-diameter high-velocity propeller is necessarily inefficient.

PROBLEMS

1. By use of the momentum theory, find the ideal efficiency of the propeller if the airspeed of the airplane is 110 mph and the slipstream is 150 mph.
2. Using the momentum theory, find the ideal efficiency of the propeller in Problem 1 if the slipstream remains the same and the forward velocity is increased to 120 mph.
3. By use of the momentum theory, find the ideal efficiency of the propeller if the slipstream is $^1/_4$ greater than the forward velocity. The forward velocity is 200 mph.

Blade-element Theory. Knowledge of the indraft velocity has enabled the propeller designer to combine with Froude's slipstream theory a theory of the action of the *elements* of the propeller blades as airfoils.

The *blade-element theory,* often called the *Drzewiecki theory,* treats the propeller action as the composite result of small elements of the propeller blade traveling along a *helix.* In Fig. 8-2a, the element of the propeller blade situated at P travels the *helical* path B and in one

turn has moved from p to p'. Still further, the helix may be unrolled and the angle β between the face of the propeller element and the plane of rotation may be denoted, Fig. 8-2b. It can be seen that, if the propeller is considered operating on an airplane flying at given speed, the angle of attack of the airfoils will be measured with respect to the new relative wind, which is a combination of the speed of rotation $2\pi Rn$ and the speed of advance $V + v/2$.

After consideration of the above conditions, it is evident that there will be only a small portion of the airfoils meeting the relative wind at the angle of attack of maximum L/D. In order to make the other elements operate under this condition, it is necessary to investigate the conditions of airflow for each section of the airfoil and twist the airfoils so that each element is set at the proper angle to the relative wind.

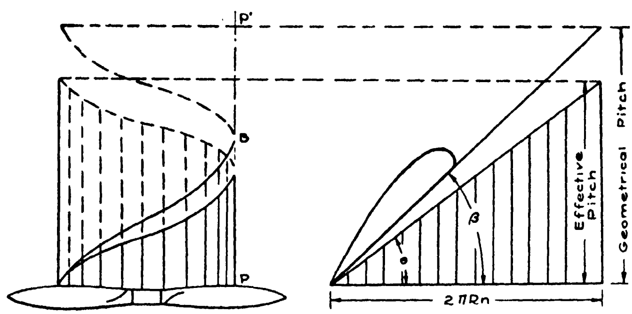


Fig. 8-2. (a) Helical path of propeller. (b) Unrolled helix path of propeller.

This problem is solved rather simply by the application of the Drzewiecki blade-element theory. These *differential* lengths of the propeller are investigated separately, and the results are summed up by a process of *graphical integration*.

Consider a certain blade element of the propeller at a radius R from the center line of the shaft. In one rotation of the propeller, this blade element travels a distance of $2\pi R$ feet around a circular path of the propeller disc. If the speed of rotation is n revolutions per second, the linear velocity of that element will be $2\pi Rn$ ft per sec. By application of the momentum theory concerning the indraft velocity, $v/2$, the sum of the forward velocity of the airplane and the indraft velocity will give a resultant forward speed of $V + v/2$ ft per sec. The resultant of the forward velocity $V + v/2$ and the *peri-*

pheral velocity $2\pi Rn$ will make an angle ϕ with the propeller element so that

$$\tan \phi = \frac{V + \frac{v}{2}}{2\pi Rn} \text{ (See Fig. 8-3)}$$

As brought out before, the angle of attack α of the blade element is set at the best L/D of the airfoil section used. Every blade element has the same forward velocity, $V + v/2$, but at every station along the span the peripheral velocity is different, *i.e.*, toward the tip the rotation speed is greater due to the increase in radius. It follows that toward the tip the angle ϕ will be less due to the increase of the resultant velocity V_R (Fig. 8-3). If the angle of attack α is to be maintained along the propeller, then

$$\alpha = \beta - \phi$$

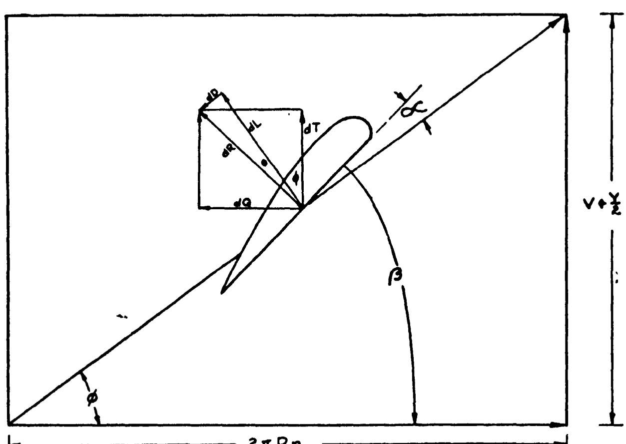


Fig. 8-3. Blade element theory.

This, of course, is the reason that the propeller is twisted.

In Fig. 8-3 the usual reaction of the airfoil is realized by the lift, dL, and the drag, dD, forces. The resultant force, dR, of the lift and drag components are resolved into two other components, namely, the *thrust*, dT, parallel to the forward velocity and the *torque force*, dQ, parallel to the plane of rotation. Let θ be the angle between the lift component and the resultant force,

then

$$\tan \theta = \frac{dD}{dL}$$

The angle θ can be found easily since the best L/D of the particular airfoil is known.

It is seen that $\cos\theta = dL/dR$, then the magnitude of the resultant force may be expressed

$$dR = \frac{C_L\ \rho/2\ cds V_R^2}{\cos\theta}$$

since

$$dL = C_L\ \rho/2\ cds V_R^2$$

where c = the chord of the blade element
ds = the differential span, then
cds = the area of the blade element

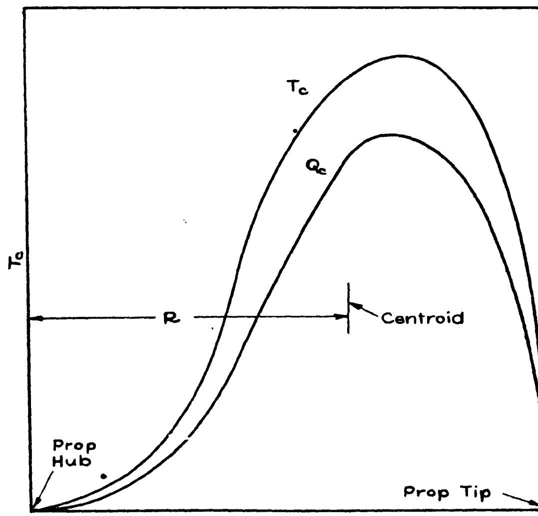


Fig. 8-4. Propeller thrust and torque curves from hub to tip.

The lift and drag forces having been resolved into forces parallel and perpendicular to the axis of the propeller shaft, the thrust given by the blade element may be expressed as

$$dT = dR\cos(\phi + \theta)$$

or

$$dT = \frac{\rho/2\ C_L cds V_R^2 \cos(\phi + \theta)}{\cos\theta}$$

The force perpendicular to the axis, called the torque force, is required to turn this blade element

$$dQ = dR\sin(\phi + \theta)$$

This process is repeated for elements every 6 in. along the span, and these quantities of thrust and torque are plotted as ordinates against the corresponding values of R as abscissas, Fig. 8-4. To

find the total thrust and the total torque forces, it is only necessary to multiply by the number of blades, since the areas under the curves are the total torque and thrust for one blade. The total torque required to turn the propeller will be the product of the resultant torque force by the moment arm. The moment arm will be the distance from the propeller axis to the *centroid* of the area under the torque curve.

The efficiency of the propeller may be calculated from the thrust and torque. The efficiency of the propeller is the ratio of the power delivered to the airplane, $(V + v/2)dT$ (force dT times distance in unit times $V + v/2$), to the power required to rotate propeller $2\pi RndQ$.

$$\eta_e = \frac{\left(V + \frac{v}{2}\right)dT}{2\pi RndQ}$$

since

$$dT = dR \cos(\phi + \theta)$$

$$dQ = dR \sin(\phi + \theta)$$

and

$$\frac{V + \frac{v}{2}}{2\pi Rn} = \tan \phi$$

then

$$\eta_e = \frac{\tan \phi dR \cos(\phi + \theta)}{dR \sin(\phi + \theta)}$$

$$\eta_e = \frac{\tan \phi}{\tan(\phi + \theta)}$$

The two airfoils most commonly used for the propeller are Clark Y and RAF-6. The RAF-6 sections are usually used for high-pitch propellers and the Clark Y sections for the low-pitch propellers. It follows, then, that the range of values for θ are between 2° and 6° depending on the value of the L/D for the airfoil section used.

Example: An airplane is equipped with a propeller whose optimum blade angle β is 26° and a Clark Y airfoil section is used at its maximum L/D of 21. Find the efficiency of the optimum blade element.

Solution: Angle of attack for max L/D = 1°. Then ϕ = 26° − 1° = 25°

From Fig. 8-3, $\tan \theta = \dfrac{dD}{dL}$

$$= \frac{1}{21} = 0.0477$$

then θ is approximately 2.6°

$$\text{Efficiency } (\eta) = \frac{\tan\phi}{\tan(\phi+\theta)}$$
$$= \frac{\tan 25°}{\tan 27.6°}$$
$$= \frac{0.4663}{0.5243}$$
$$= 89\%$$

In the above example, reference was made to the *optimum blade angle,* usually considered to be at 75% tip radius. The above efficiency of 89% represents the efficiency of the blade element at that point. Out, toward the tip, or inward, toward the axis, the blade elements will not be as efficient; so the total efficiency of the propeller will be slightly less than that for the optimum blade element.

PROBLEMS

1. An airplane has a propeller whose optimum blade angle is 21° and the airfoil section used has a maximum L/D of 23. Find, by use of the blade element theory, the efficiency of the optimum blade element if α is 2° for max L/D.
2. Find, by blade-element theory, the efficiency of the optimum blade element if the L/D of the airfoil is 18, the blade angle is 34° and α is 4° for max L/D.
3. Find the efficiency of the optimum blade element if the L/D is 22.5 and the blade angle is 43°. (α is 3° for max L/D.)

Propeller Coefficients. In the N.A.C.A. Technical Reports Nos. 14 and 30, the thrust and torque coefficients are found experimentally. The other method, as described above, is by the graphical integration of the effects of the blade elements.

The angle of attack, α, has a direct relation to the lift and drag forces of the blade element, and in turn the lift and drag are resolved into the thrust and torque. The useful power derived from the propeller is thrust times the velocity and the power input is $2\pi RndQ$. It follows that since thrust and torque depend on α, they will also depend on the ratio $V/2\pi Rn$ or

$$T \text{ and } Q = f\left(\frac{V}{nD}\right)$$

The *torque* and *thrust coefficients* are defined by the N.A.C.A. as follows:

$$T_C = \frac{T}{\rho n^2 D^4}$$

$$C_Q = \frac{Q}{\rho n^2 D^5} \quad \text{where } n = \text{r.p.s.}$$

It has been found more practical to use power instead of torque, and since power is $p = 2\pi RnQ$ the *power coefficient* has been defined as

$$C_P = \frac{P}{\rho n^3 D^5}$$

Many practical propeller problems involve the diameter, which is either unknown or to be determined. A useful way to write the above expression and eliminate diameter is to substitute for C_P the quantity $K_S(V/nD)^5$ where K_S, like C_P, is also a function of (V/nD);

therefore
$$K_S \frac{V^5}{n^5 D^5} = \frac{P}{\rho n^3 D^5}$$

$$K_S = \frac{P n^5 D^5}{V^5 \rho n^3 D^5}$$

$$= \frac{P n^2}{V^5 \rho}$$

In the various N.A.C.A. tests on metal propellers, a speed-power coefficient, C_S, has been determined which is equal to $\sqrt[5]{1/K_S}$

therefore
$$C_S = V \sqrt[5]{\frac{\rho}{P n^2}}$$

This expression for the speed-power coefficient is usually written for sea-level conditions

$$C_S = \frac{0.638\,V}{P^{1/5} N^{2/5}}$$

where V = velocity of the airplane in mph
P = power in HP
N = rotation speed of propeller in rpm

Geometrical Similar Propellers. For a given propelller, the thrust and power coefficients and the efficiency may be potted against V/nD. An examination of Fig. 8-5 shows that for one particular value of V/nD the efficiency is maximum; this value of V/nD which gives the maximum efficiency is called the design, V/nD.

The plot of the propeller curves such as Fig. 8-5 will apply to all such propellers which are geometrically similar.

Pitch Ratio. A propeller advances as it rotates. This actual advance, in one revolution, is the *effective pitch* of the propeller. The path of any section of a propeller blade is a helix. The *geometrical pitch* of an element is the distance which the element would advance along its axis of rotation if it were moving along a helix whose slope was that of the blade angle. The geometrical pitch of the propeller

is usually stamped on the propeller blade. The *nominal* or *standard* geometrical pitch was, at one time, the pitch of the section at 75% R. Recently most metal adjustable-pitch propellers are designed with the standard or nominal pitch at the 42-in. radius. This fixed radius has been chosen in place of the 75% R because it is the practice to change the diameter of standard-made propellers by cutting off the tips, this would greatly affect the efficiency of the 75% R but would not affect the 42-in. radius.

The *diameter* of the propeller is the distance from tip to tip; the *radius* R is one-half this diameter. The ratio of pitch to diameter has been called the *pitch ratio* of the propeller.

The difference between the *effective pitch* and the geometrical pitch is called the *slip*. This slip is the measure of the efficiency of the propeller, *i.e.*, the less the slip the greater the *efficiency*.

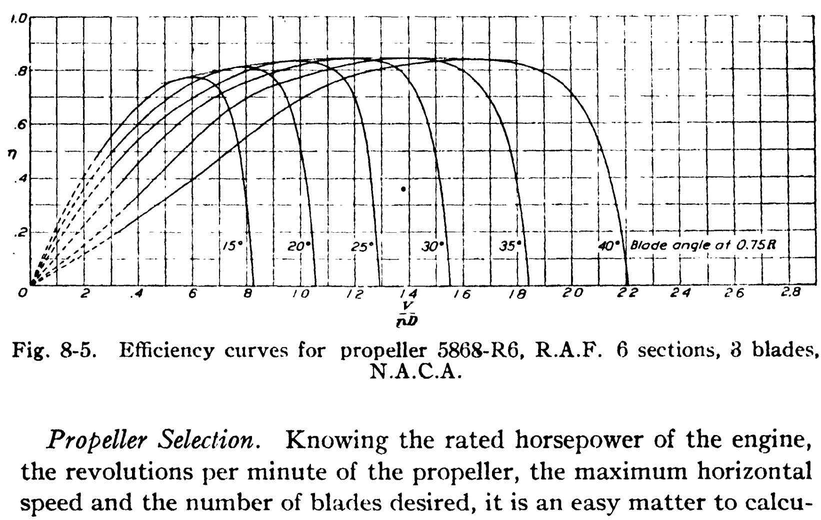


Fig. 8-5. Efficiency curves for propeller 5868-R6, R.A.F. 6 sections, 3 blades, N.A.C.A.

Propeller Selection. Knowing the rated horsepower of the engine, the revolutions per minute of the propeller, the maximum horizontal speed and the number of blades desired, it is an easy matter to calculate the diameter, pitch, and efficiency of the propeller. The N.A.C.A. Reports 350 and 640 give working charts and directions which facilitate the selection of propellers.

To select the correct diameter and pitch of a propeller it is first necessary to calculate the value of the power-speed coefficient from the horsepower, revolutions, forward speed, and altitude at which the propeller is to operate. Next, the pitch setting for the propeller operating at the desired portion of the efficiency curve is chosen, depending upon the calculated C_S and the airplane performance desired. Then, from the lower curves, the value of V/nD for the calculated C_S and the corresponding blade angle is found. The diameter may be calculated after finding the values of V/nD, n, and V. If for

any reason the diameter of the propeller has already been decided upon, this gives the V/nD a fixed value also. The pitch setting can easily be found directly from the curves of V/nD versus C_S.

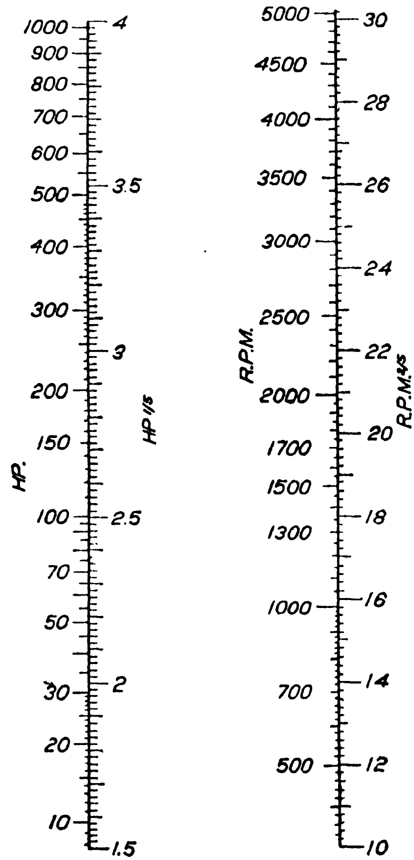


Fig. 8-6. Scale for finding $HP^{1/5}$ and $RPM^{2/5}$, N.A.C.A.

Example: It is desired to find the diameter and pitch for a particular 2-bladed propeller. The rated HP of the engine is 220 HP at 1700 rpm, the propeller is to be designed for maximum efficiency at an airspeed of 130 mph at sea level.

Solution:

$$C_S = \sqrt[5]{\frac{\rho V^5}{Pn^2}}$$

For standard conditions $C_S = \dfrac{0.638 \times mph}{HP^{1/5} \times rpm^{2/5}}$

From Fig. 8-6 find the fifth root of 220 HP and the 1700 rpm to the two-fifths power.

$$C_s = \frac{0.638 \times 130}{2.94 \times 19.6}$$

$$= 1.44$$

Unless otherwise specified, it is always assumed that the propeller is to operate at its maximum efficiency at the high speed of the airplane. Then from the upper or efficiency curves, Fig. 8-7, it will be

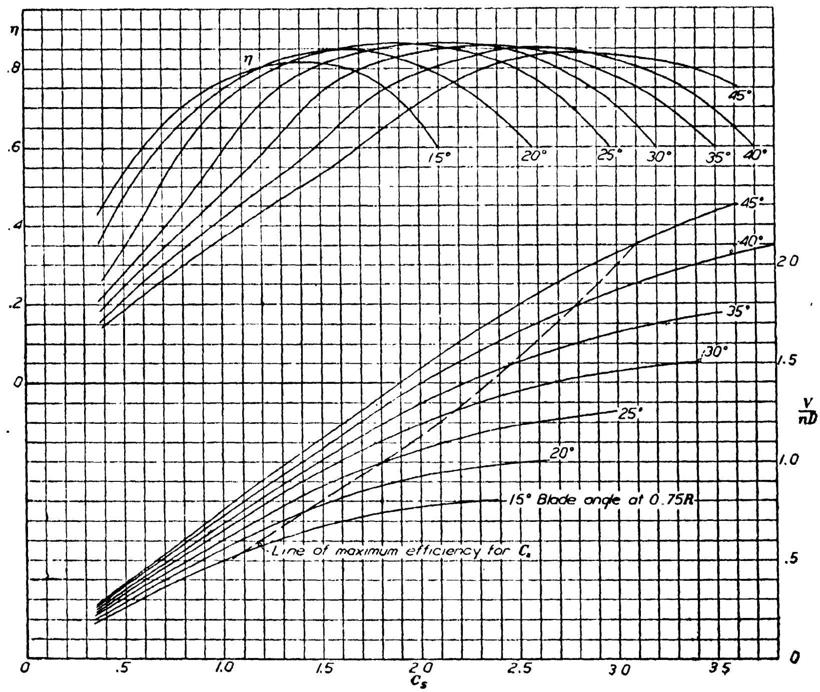


Fig. 8-7. Design chart for propeller 5868-9, Clark Y section, 2 blades, N.A.C.A.

seen that a blade setting of 20° at 0.75 R satisfies this condition, *i.e.*, the efficiency for a setting of 20 degrees is maximum for C_S of 1.44.

From the lower set of curves in Fig. 8-7, for C_S of 1.44 and a blade angle of 20 degrees, $V/nD = 0.76$. Then the required diameter is

$$D = \frac{V \times 88}{rpm \times (V/nD)}$$

$$= \frac{130 \times 88}{1700 \times 0.76} \quad \text{where 88 is a conversion factor}$$

$$= 8.85 \text{ ft}$$

The propulsive efficiency, from the upper curves is 0.84. This is for a 2-blade propeller with a Clark Y airfoil section. The propeller thus selected will give the greatest possible high speed, and is sometimes referred to as a speed propeller, though, also, it is nearly the best propeller for climb, and has been referred to as a best-performance propeller. For best ceiling use the lower dashed line in Fig. 8-7. This dashed line in the lower curves has been drawn in and is called the line of maximum possible efficiency for the design C_S. This means that for the above C_S of 1.44 the line for *maximum ceiling* determines the blade angle to be about 21°. (This angle of 21° is the angle in operation at 0.75 R and includes deflection. This deflection will be approximately $^1/_4$ degree at 220 Hp, so that the setting under static conditions will be 20.75°.)

Then from the lower curves, at a C_S of 1.44 and a blade angle of 21 degrees, the V/nD is 0.78. For the best ceiling, that is, for a "peak-efficiency" propeller the corresponding diameter would be

$$D = \frac{130 \times 88}{1700 \times 0.78}$$
$$= 8.63 \text{ ft}$$

The propulsive efficiency would be approximately 0.85. These values for angle of attack, between 20° and 25° and the corresponding efficiency, make it necessary to interpolate because they have not been included in the above plot. However, in a case like that above, decreasing the diameter of the propeller will necessitate a greater blade angle and also will produce a greater efficiency. A higher value of efficiency can always be obtained at the same value of C_S by use of the line of maximum efficiency for C_S.

It is sometimes necessary, because of the design of the airplane, to assign a value for the diameter of the propeller and, from the horsepower and rpm of the engine, to calculate the best blade angle and the efficiency at maximum speed.

Example: Suppose in the above example, where HP is 220 and rpm is 1700, a 9-ft propeller is to be used. Find the blade angle and efficiency at the maximum horizontal speed.

Solution: Calculate

$$\frac{V}{nD} = \frac{88 \times mph}{rpm \times D}$$
$$= \frac{88 \times 130}{1700 \times 9}$$
$$= 0.748$$

The C_S = 1.44 as before, then, from the lower curves in Fig. 8-7 for

V/nD of 0.748 and C_S of 1.44 the blade angle at $0.75\ R = 19°$. From the upper curves of Fig. 8-7 at $C_S = 1.44$, and blade angle 19°, the efficiency = 0.84.

It is reasonable to consider that an increase in diameter of the propeller makes for greater efficiency, since

Thrust = Mass of air handled per second × difference between free stream velocity and slipstream velocity. Also, the greater the mass of air, the lower the slip.

This seems to be misleading, since the efficiency of the 9-ft propeller in the above problem was calculated to be less than the propeller of 8.63 ft. The reason the smaller diameter propeller is more efficient is the fact that the dashed line of maximum possible efficiency was used, whereas it was not used for the 9-ft propeller. The length of the propeller diameter is limited, due to dimensions of the airplane, tip speed of the propeller, and stresses in the propeller.

The N.A.C.A. Technical Report No. 351 reveals how a standard, adjustable-blade, aluminum-alloy propeller was tested for efficiency drops at different diameters. The diameter of the propeller blade was originally 10 ft and was cut off successively by taking off 3 in. from each tip and then rounding the edges by a circular arc tangent to the leading and trailing edges (see Fig. 8-8). The propeller, in each successive test from 10 ft to 8 ft in diameter, showed corresponding successive losses in efficiency. The total loss from the maximum efficiency was only 6%, that is, from 10 ft to 8 ft. Changes in pitch will not appreciably affect this result. The important conclusion reached is that a propeller, adapted to a particular engine and airplane, which has its tip cut off and rounded will be only slightly less efficient than a specially designed propeller. However, it is evident that this cutting-off process can be carried too far.

PROBLEMS

1. The rated HP of an engine is 450 HP at 1800 rpm. If the design airspeed is 180 mph at sea level, find, for a 2-blade propeller: (a) the power-speed coefficient; (b) blade angle at 0.75 radius; (c) the required diameter.

2. An airplane has a 300-HP engine at 2000 rpm. If the design airspeed is 165 mph, at sea level, find, for a 3-blade propeller: (a) the power-speed coefficient; (b) blade angle at 0.75 radius; (c) the required diameter: (d) the propulsive efficiency.

3. An airplane has a 1000-HP engine at 1900 rpm. If the design airspeed is 240 mph at sea level, find for a 4-blade propeller: (a) the power-speed coefficient; (b) blade angle at 0.75 radius; (c) the required diameter; (d) the propulsive efficiency.

4. **Find, for maximum possible efficiency (maximum ceiling) the blade-angle setting at 0.75 radius for an airplane with an engine rated**

at 180 HP at 2100 rpm at sea level. The design speed is 145 mph. What is the diameter for this peak-efficiency propeller of 2 blades?

5. An airplane with design speed of 175 mph has an engine rated at 250 HP at 1900 rpm. If the diameter of the propeller is fixed at 8.8 ft, find the blade angle and efficiency at the design speed for a 2-blade propeller.

Fig. 8-8. Propeller series of five diameters.

6. In Problem 5 (above), the diameter due to necessary ground clearances was reduced to 8 ft. If the HP, design speed, and rpm remain the same, find the blade angle and efficiency for a 2-blade propeller.

7. An airplane with a design speed of 130 mph has an engine rated at 125 HP at 2100 rpm. If the diameter of the propeller is

fixed at 7.5 ft, find the blade angle and efficiency at the design speed for a 2-blade propeller.

8. An airplane has a 145-HP engine at 1850 rpm. If the design airspeed is 120 mph at sea level, find for a 2-blade propeller: (a) the power-speed coefficient; (b) blade angle at 0.75 radius.

9. In Problem 8, find the required diameter and the propulsive efficiency.

10. An airplane with a design speed of 100 mph has an engine rated at 90 HP, at 2100 rpm. If the diameter of the propeller is fixed at 7.3 ft, find the blade angle and efficiency for a 2-blade propeller.

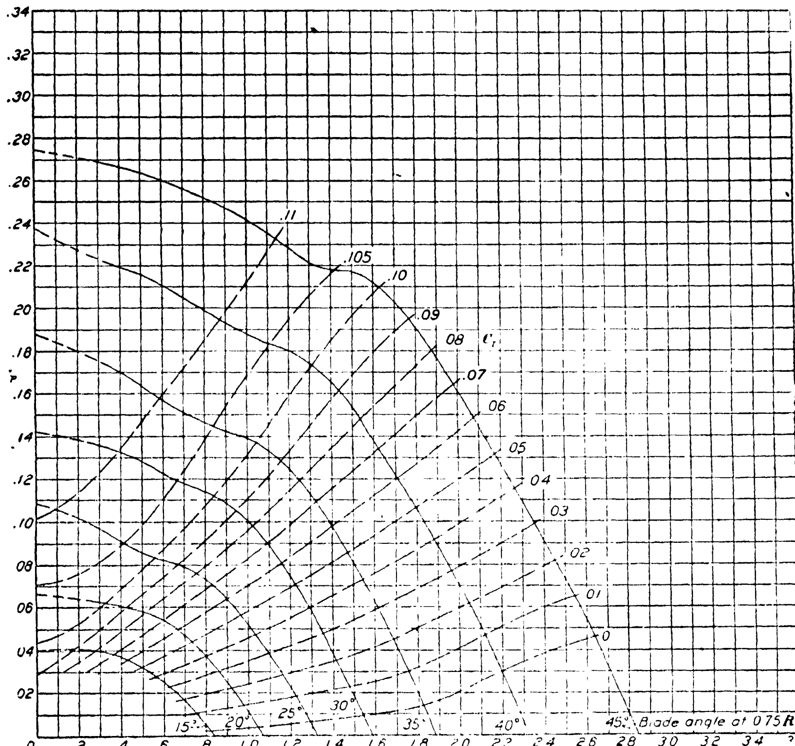


Fig. 8-9. Power-coefficient curves for propeller 5868-9, Clark Y section, 2 blades.

Calculation of Propeller Thrust. The N.A.C.A. Technical Report No. 640 on characteristics of full-scale propellers of 2, 3, and 4 blades, presents a method for calculating thrust for a fixed pitch propeller and an illustrated example. The following has been quoted from the appendix of the report.

"It is assumed in the use of the method of calculating the thrust for fixed-pitch propellers that the following sea-level design characteristics of the airplane, the engine, and the propeller are known:

V_0, design airspeed, mph

N_0, design engine speed, rpm

$(b.hp.)_0$, design engine power (rated power)

$\left(\frac{V}{nD}\right)_0 = J_0$, design $\frac{V}{nD}$

η_0, design efficiency (high speed or cruising)

D, propeller diameter, ft

β_0, design blade angle at 0.75 R

"The method may conveniently be put into step form as follows:

(1) Using J_0 and β_0, obtain C_{T_0} and C_{P_0} from charts of C_T and C_P against V/nD, Fig. 8-9.

(2) At even values of J pick off values of C_T and C_P along line of constant β_0 (interpolate when necessary).

(3) Compute $\frac{J}{J_0}$, $\frac{C_T}{C_P}$, $\frac{C_{P_0}}{C_P}$

(4) Compute $\frac{N}{N_0} = \sqrt{\frac{C_{P_0}}{C_P}}$

(5) Compute $T_0 = [\eta \times (b.hp.)_0 \times 375]/V_0$

(6) Compute $V = V_0 \times \frac{J}{J_0} \times \frac{N}{N_0}$

(7) Compute thrust $T = T_0 \times \frac{C_{P_0}}{C_{T_0}} \times \frac{C_T}{C_P} = K \times \frac{C_T}{C_P}$

where $K = T_0\, C_{P_0}/C_{T_0}$

This method assumes that the full-throttle engine torque is constant."

As an example, assume that it is desired to obtain the propeller thrust through the take-off and climbing ranges for an airplane having the following characteristics:

$V_0 = 190$; $N_0 = 1500$; $(b.hp.)_0 = 600$; $J_0 = 1.00$; D_0(2-blade) = $11'\ 1^1/_2''$; $\eta_0 = 0.862$; $\beta_0 = 25°$

Blade section = Clark Y

The computed data may be conveniently tabulated as follows:

$C_{T_0} = 0.0448$; $C_{P_0} = 0.0520$;

$T_0 = 1020$ lb; $J_0 = 1.00$

TABLE III

J	J/J_0	C_T	C_P	C_T/C_P	C_{P_0}/C_P	N/N_0	V (mph)	T (lb)
0.1	0.1	0.110	0.1056	1.042	0.493	0.702	13.3	1232
0.2	0.2	0.1075	0.1017	1.059	0.512	0.720	27.4	1252
0.3	0.3	0.1058	0.0972	1.088	0.535	0.731	41.7	1287
0.4	0.4	0.1055	0.0911	1.158	0.571	0.755	57.4	1370
0.5	0.5	0.1037	0.0858	1.207	0.607	0.778	74.0	1427
0.6	0.6	0.0970	0.0823	1.178	0.632	0.795	90.6	1392
0.7	0.7	0.0870	0.0790	1.100	0.658	0.811	108.0	1300
0.8	0.8	0.0748	0.0732	1.022	0.710	0.842	128.0	1210

$$K = \frac{0.0520}{0.0448} \times 1020 = 1182$$

In the above explanation it can be seen that the selection of even values of V/nD from 0.1 through 0.8 represents certain airspeeds in

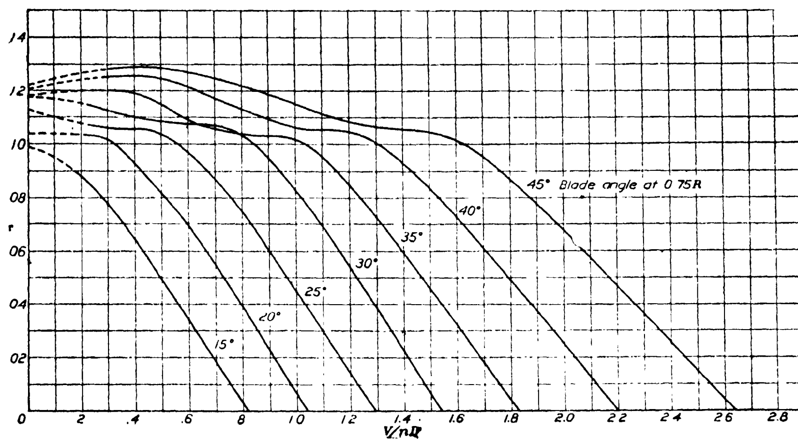


Fig. 8-10. Thrust-coefficient curves for propeller 5868-9, Clark Y section, 2 blades.

the take-off range; namely, 13.3 mph through 128 mph. The same method may be used for the climbing and flight range. When the airplane is not moving, V is zero, which makes V/nD zero. At this condition, the thrust and power coefficients have large positive values. The magnitude of the thrust coefficient when $V/nD = 0$ is a measure of the ability of the airplane to accelerate from a standstill position. It can be seen from the above data that as the V/nD increases the thrust and power coefficients decrease. It can also be seen from the plot of thrust coefficient versus V/nD, Fig. 8-10, blade setting of 25°, that when the forward velocity of the airplane increases to give a V/nD value that exceeds 1.3, the thrust coefficient is zero. This means that the relative wind is striking the blade element at the

angle for zero lift. That is, the propeller is advancing the greatest possible distance per revolution. Any further increase in forward speed will produce a thrust force acting backward, *i.e.*, a negative thrust.

Number of Blades. Until recently one could rarely find an airplane equipped with more than two blades. The reason was that the 2-blade propellers are lighter, cheaper, and generally thought of as being more efficient. With the ever-increasing number of high-powered engines the propeller blades were increased in number. Three or four blades are used if the diameter is limited to a particular size, or where the operation is not smooth with a 2-blade propeller due to body interference or the slipstream of another propeller. In the very large and slowly turning propellers the vibrations may become so great on 2-blade propellers that, to remedy this condition, it is necessary to use 3 or more blades. The 2-blade propeller in larger installations also has been found to be overloaded in yawing and this will cause pulsations that can be corrected by additional blades.

In Figs. 8-11 and 8-12 are found design charts for 3- and 4-blade propellers taken from N.A.C.A. Technical Report No. 640. This particular test was to find the difference in efficiency of the 2-, 3-, and 4-blade propellers. The group of propellers tested was composed of propellers having RAF-6 and Clark Y sections.

It was found that:

(1) increasing the solidity by adding blades had a lesser adverse effect than increasing it by increasing the blade width;

(2) the loss in efficiency commonly conceived to be the result of increasing the number of blades was not fully realized, only about two percent difference in peak efficiency between a 2-blade and a 4-blade propeller being measured; and

(3) an increase in solidity tended to delay the stall and to increase the efficiency in the take-off range.

Blade Width. The propeller blade *width ratio* (WR) is the ratio of the width of the propeller blade, at the nominal or standard blade element, to the diameter. The nominal blade element has been referred to as the element at 0.75 R; in metal propellers the nominal blade element is at 42 in. from the axis. The *total width ratio* (TWR) is the width ratio times the number of blades. The total width ratio, independent of the number of blades or the width of each, is the factor affecting efficiency. Modern design for metal propellers has a blade width ratio of approximately 0.053, and for very narrow blades the width ratio is about 0.02. For the narrow blades the efficiency is slightly higher. The above methods for selecting propellers assumed that the propeller under consideration had the same blade width as

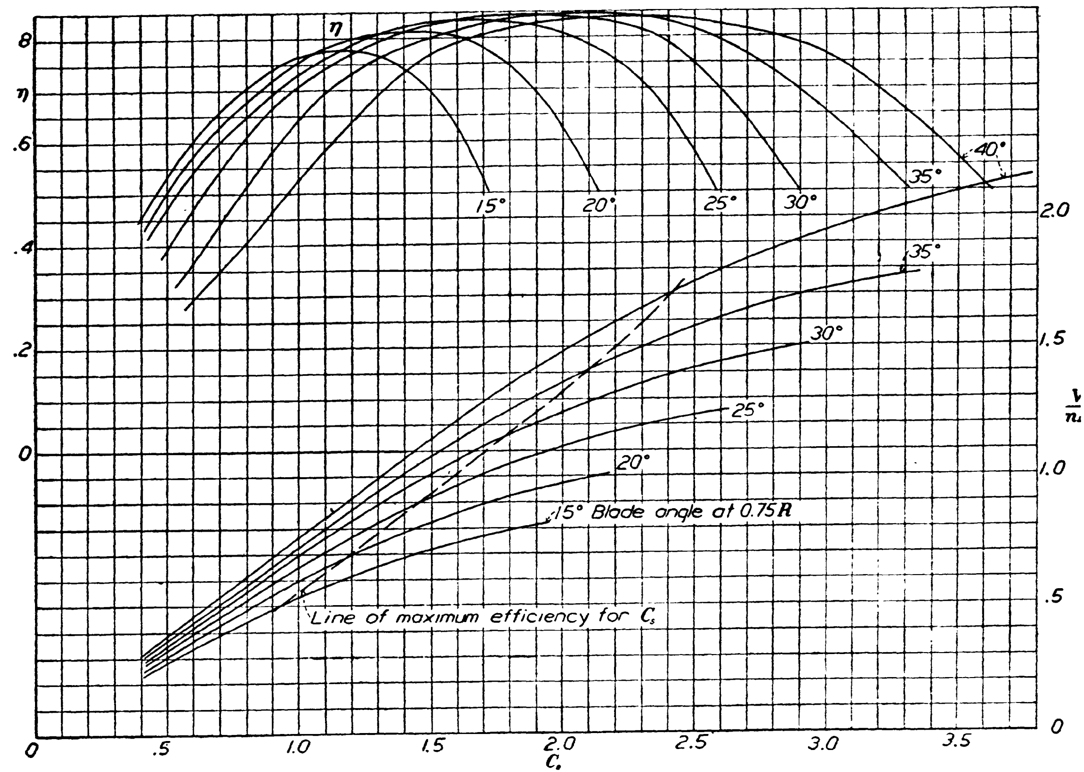


Fig. 8-11. Design chart for propeller 5868-R6, R.A.F., 6 sections, 3 blades, N.A.C.A.

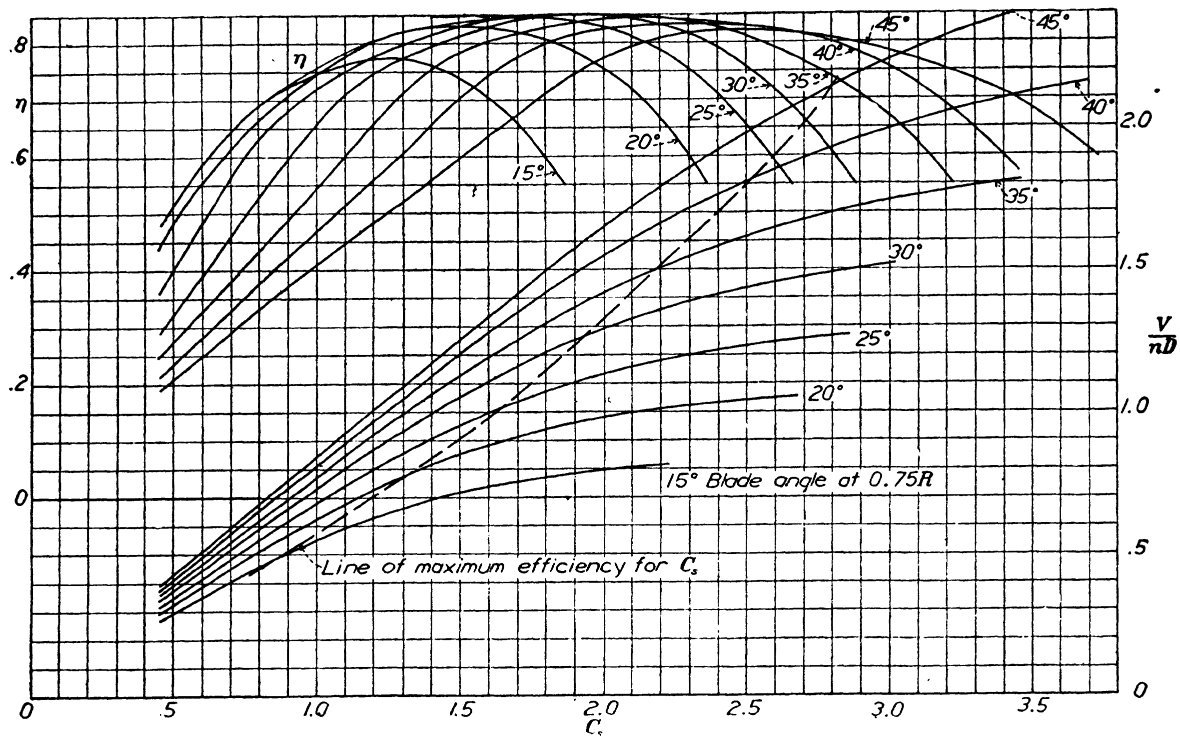


Fig. 8-12. Design chart for propeller 5868-9, Clark Y section, 4 blades, N.A.C.A.

the ones for which the data are given in the N.A.C.A. reports. It may be desired to find the diameter, the design blade-angle setting, and the thrust of a propeller having a different blade width from those tested. As was shown earlier the expressions for power and thrust were resolved from the lift and drag forces which are dependent on blade areas (or blade widths). Therefore in the calculation of C_S, the power should be multiplied by the ratio of the blade widths, b_1/b_2, where b_1 is the blade width at the nominal blade element of the propeller for which the design charts were made and b_2 is the blade width at the same radius for the propeller under consideration.

This same ratio should be used to calculate the value of C_P to be used in obtaining the take-off thrust, and the take-off thrust obtained

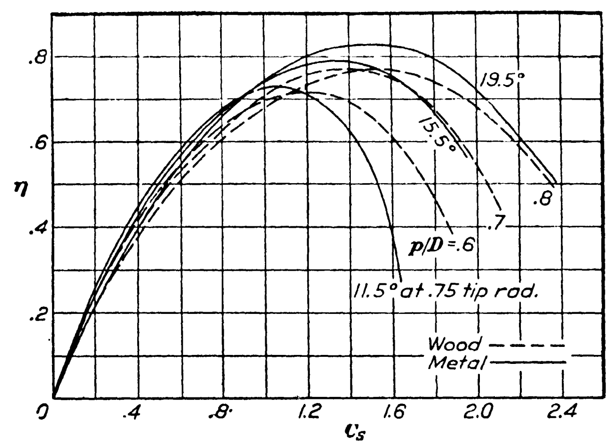


Fig. 8-13. Comparison of efficiencies of wood and metal propellers, N.A.C.A.

from the charts should be divided by this ratio to obtain the actual thrust of the propeller.

Thickness Ratio of the propeller is another factor affecting the efficiency. This is the ratio of the blade thickness at the nominal element (0.75 R or 42 in.) to the blade width at that element. The usual thickness ratios for the metal propellers are about 0.07 and for wooden propellers the thickness ratios are on the order of 0.13.

The reason for the differences in the thickness ratios of wooden and metal propellers can best be understood by the stresses in a propeller in flight:

(1) The bending moment due to the air forces.
(2) The centrifugal forces due to the rotation of the blades.
(3) Torsional and bending stresses imposed by the gyroscopic action when in turning or yawing motions.

This is not a complete list of the static loads but it will give some idea as to the magnitude of the required strength for all propellers. For the same strength, the metal propeller can be made with a thickness ratio less than that for a wooden propeller and the actual efficiency of the metal propeller is greater due to the thinner blades. It has been found that for the thinner metal propellers with either RAF-6 or Clark Y sections the efficiency is from 4 to 7 percent better than wooden propellers (see Fig. 8-13).

The fabrication process for the metal propeller is more expensive than that for the wooden propeller but the metal propeller can be repaired for cracks or bends whereas the wooden propeller must be replaced.

Aspect Ratio for Propellers. The blade form for propellers is usually determined for a given material or service. For metal propellers on high-speed airplanes, and with high rated rpm the aspect ratio can be greater because of the strength of the material. For metal propellers, the *aspect ratio* can be as high as 8 or better and with these higher aspect ratios the efficiency is greater. On slower airplanes, with lower rated rpm, better efficiency can be obtained with propellers of lower aspect ratio and a wooden propeller may be the most appropriate, with aspect ratio ranges from 4 to 6. The aspect ratio of 4 is about the lowest ever found in propeller design and for wooden propellers an aspect ratio of 6 is rarely ever exceeded because the sections near the tip are not strong enough to withstand the imposed loads.

Planform. As shown in the chapter on induced drag, the tapered wings with rounded tips reduce the wing tip vortices and in turn increase the efficiency. This same theory exists in propeller design (see Fig. 8-14). The propeller blade is more efficient if the ends are tapered and the tips are rounded. However, the particular shape or amount of taper has little or no effect on the efficiency and, although no particular increase in efficiency was noted for sweep-back and rake, the bending moments due to centrifugal force were eliminated.

Interference. The N.A.C.A. Technical Report No. 642 gives a complete account of tests of five full-scale propellers in the presence of a radial and a liquid-cooled engine nacelle. Almost all the propeller work charts published by the N.A.C.A. have already taken into account the body interference. The charts included in this chapter are from tests using a smooth liquid-cooled engine nacelle in the presence of the various propellers. The *interference* for most engine-nacelle combinations tends to reduce the propeller efficiency on the order of 3 percent.

If a designer wished to use the charts for a propeller that has to be

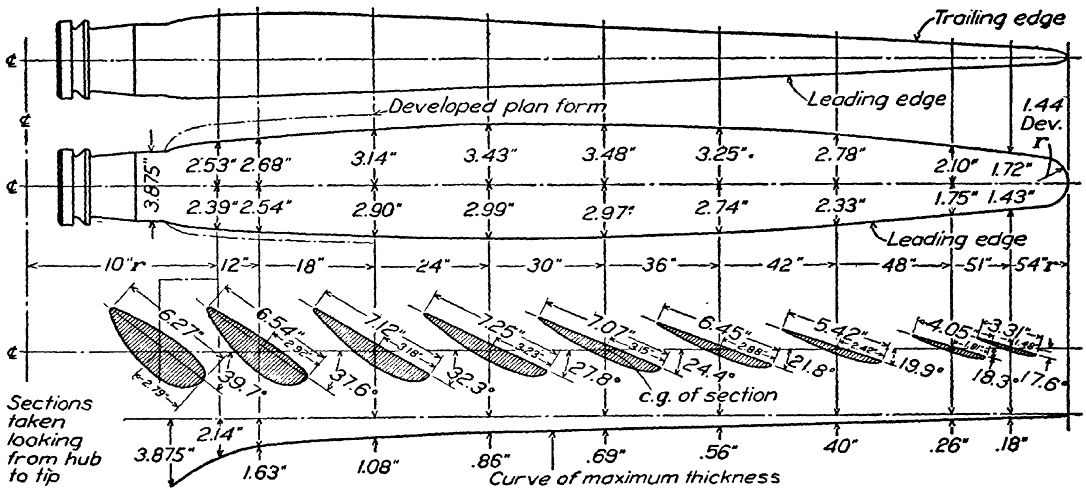
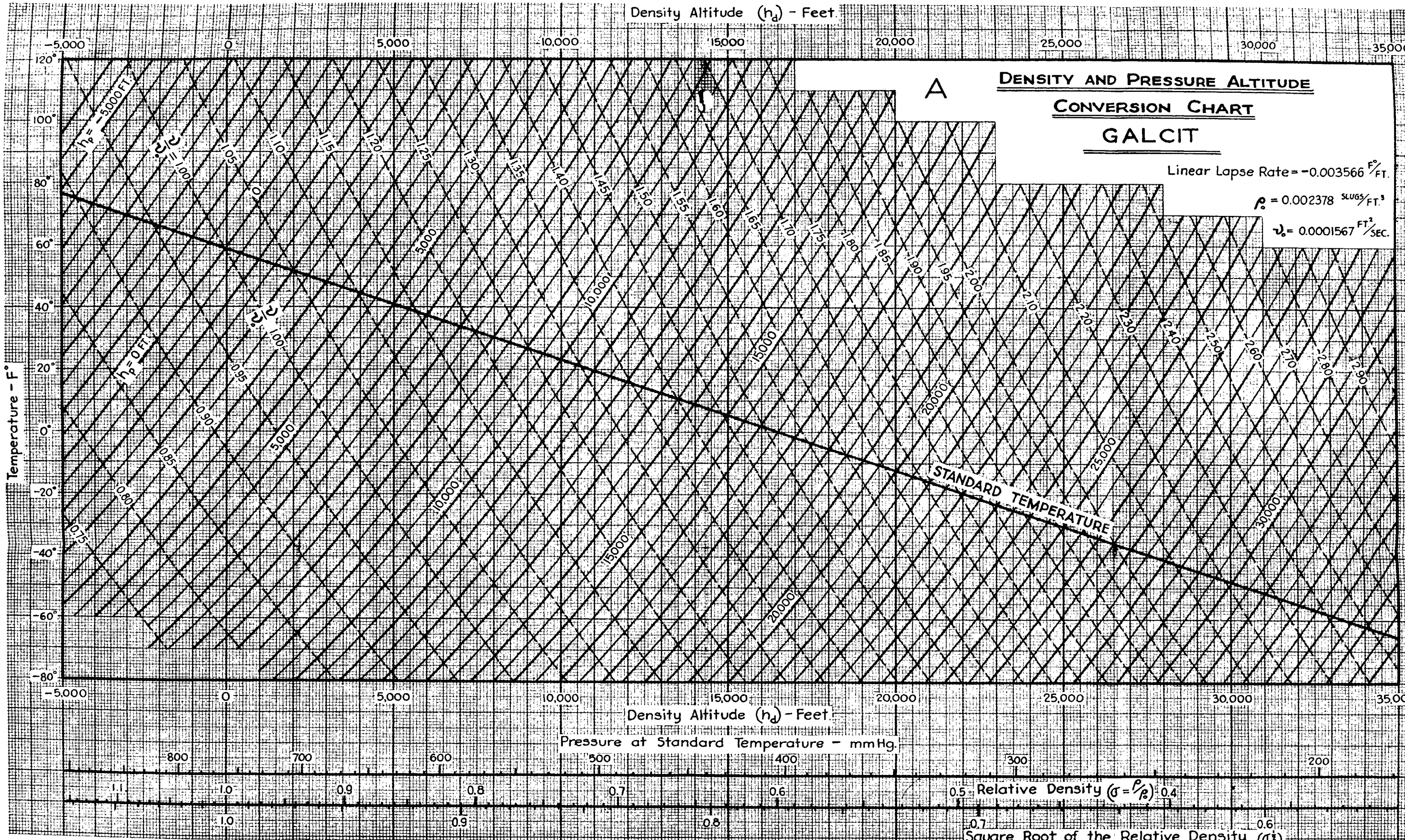


Fig. 8-14. Propeller planform, N.A.C.A.



A
DENSITY AND PRESSURE ALTITUDE
CONVERSION CHART
GALCIT
Linear Lapse Rate = −0.003566 F°/FT.
ρ₀ = 0.002378 SLUGS/FT.³
ν₀ = 0.0001567 FT.²/SEC.
Density Altitude (h_d) - Feet.
Temperature - F°
STANDARD TEMPERATURE
h_p = −5,000 FT.
h_p = 0 FT.
ν/ν₀ = 1.00
Pressure at Standard Temperature - mm Hg.
Relative Density (σ = ρ/ρ₀)
Square Root of the Relative Density (σ^½)

used in the presence of a nacelle which is greatly different from the liquid-cooled engine which the original tests were made, corrections would be necessary. Information regarding effects of the body on the propulsive efficiency may be found in the N.A.C.A. Technical Report No. 642, 1938. In performance consideration, which will be covered to some extent in a later chapter, the added drag of the portions of the airplane (other than the nacelle) should also be taken into account. The drag of parts of the airplane that lie in the slipstream, such as tail surfaces, landing gear, etc., may be approximated from the following relation:*

$$\frac{\Delta D}{D} = 2.5C_T/J^2$$

where

$$\begin{aligned} \Delta D &= \text{added drag} \\ D &= \text{drag without slipstream} \\ J &= V/nD \end{aligned}$$

Propeller reduction gears have been used where the propeller is mounted considerably ahead of the engine. In the N.A.C.A. Report No. 338 propulsive efficiencies from 35 to 75 percent have been reported, due entirely to reduction in propeller-body interference.

Reduction gearing is considered necessary only to avoid high tip-speed losses. Because of the added weight which usually offsets the improved performance, this practice is not generally followed in any of the commercially designed airplanes. It is conceded, however, that the propeller reduction gear does give greater speed and improved take-off and climb characteristics.

Materials of Construction. Propellers have been made successfully from three basic materials. These materials are wood, bakelite, and metal. For wooden propellers the following woods are preferred: (1) walnut, (2) birch, (3) oak. A wooden propeller is made up of several laminations glued together, each layer of wood extending from one blade to the other. The propeller is finished with a moisture-repellent varnish and then the blade tips are covered with thin sheet brass. Wooden propellers are thicker and hence are not so efficient as metal or bakelite propellers. Because cost of construction is lowest for wooden propellers, the majority of the light airplanes are equipped with fixed-pitch wooden propellers.

The bakelite, sometimes called Micarta, propeller is made of layers of stiff canvas and bakelite, constructed in one continuous piece and formed by heavy pressure in a mold. This type of propeller is less affected by moisture than the wooden propeller and, whereas the wooden propeller may be damaged in rain, sand, etc., the bakelite

* N.A.C.A. Technical Report No. 640, 1938.

material will withstand adverse conditions much better. The bakelite propeller is the heavier material and is more expensive to make.

Aluminum has found its way into the construction of metal propellers because of its efficiency, durability, strength, and ease of fabrication. Hollow, steel, seamless propellers also have been found to be very satisfactory. In the case of an accident resulting in damage to the propeller, the metal propeller can be reconditioned and put back into service, while wood and bakelite propellers must be discarded.

Propeller Classification. There are, in general, four classifications of propellers with respect to their adjustment of pitch:

(a) The Fixed-pitch Propeller is usually considered as one in which the whole propeller is in one solid piece, *i.e.*, the blade setting is not adjustable. The fixed-pitch propeller is made to give its maximum efficiency at one engine speed and one cruising speed, in other words, the fixed-pitch propeller incorporates one particular design V/nD. Except for racing planes, where maximum top speed is desired, the fixed-pitch propeller is usually designed to give maximum efficiency at the design cruising speed. Where a high-pitch propeller is used, the engine speed on the take-off is held down and hence the take-off performance is very poor.

(b) Adjustable-pitch Propellers were developed because of the unsatisfactory performance of the fixed-pitch propeller. The first adjustable propeller had to have the pitch changed on the ground while the engine was inoperative. This type was not completely satisfactory because of the time and effort required to change the blade angles, but it did give the pilot at least two propellers in one. In low pitch (small helix angle) the propeller would give good performance in take-off and climb, but the engine would tend to race in level cruising attitude, and therefore could not make full use of the horsepower available. In high pitch (large helix angle), just as in a *club* propeller used for engine testing, the revolutions of the engine are held down and this prevents the development of the horsepower required for proper take-off. The take-off run is prolonged with the higher pitch but a greater top speed can be realized in level flight.

(c) The Controllable-pitch Propeller has been so designed that the pilot may manually change the propeller pitch for the desired performance of climb or cruising, while the engine is in operation. The first of the controllable propellers was made for only two operations, either low or high pitch, *i.e.*, for take-off and climb or cruising. The later and more controllable propellers gave a wide range of pitch from very flat pitch to *full feathering*. Full feathering of a propeller means that the blades are turned to the position of minimum drag. If the engine fails in flight, the blades can be turned to full feathering and

thus keep the engine from "windmilling." This is of major importance in minimizing the possibility of damage to the engine or airplane when engine trouble occurs.

(d) The Constant-speed Propeller is one of the more recent inventions making possible propeller operation at maximum efficiency for all flight conditions.

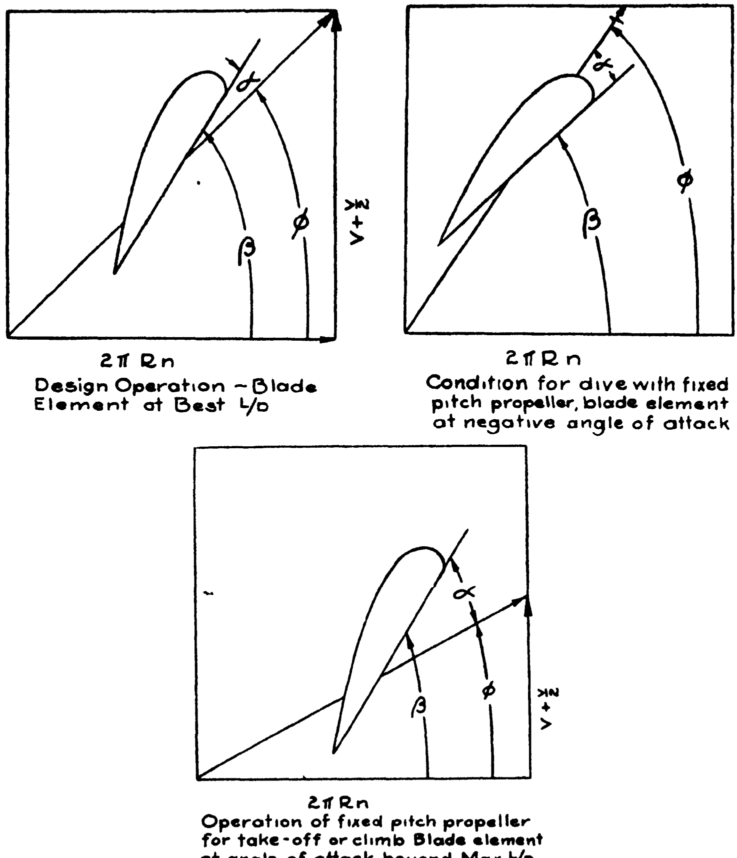


Fig. 8-15. Blade element theory for constant speed propeller.

A propeller whose angle β is fixed, as in the fixed-pitch propeller, will give its maximum efficiency in only one condition of flight, *i.e.*, the nominal blade element gives its maximum efficiency at only one value of V/nD. Figure 8-15 shows the reason for this: If the value of V (forward velocity) is changed, either by take-off or climb, etc., the value of the angle ϕ will be changed. In the case of the fixed-pitch propeller the angle β is always the same. The angle α is the difference

between the angles β and ϕ, and is the angle for the best L/D of the blade element. If the forward velocity, V, is changed, the angle ϕ will change and therefore the angle α will necessarily be changed. That is to say: if the airplane is in a climb, the forward velocity will be small and the angle ϕ will be reduced. If the angle ϕ is reduced, the angle α will be increased, and the airfoil will not be operating at its best L/D, *i.e.*, the drag on the blades will be greater. Likewise, if the airplane is in a dive, the velocity, V, will be greater; ϕ will be increased and α will be decreased. If the dive is fast enough, it may decrease the angle of attack beyond that for zero lift and actually cause a resultant force acting backward.

The above-mentioned problems have been solved by the use of the constant-speed propellers. With these propellers the pilot is able to change the angle β in the air and so keep the efficiency of the propeller constant over practically all ranges of speeds. There are, generally speaking, two types of constant-speed propellers: the electrically controlled and the hydraulically controlled. With these two types it is only necessary for the pilot to set the throttle to the desired crankshaft speed and the controlling mechanism automatically sets the propeller blade angle at the proper angle for best performance.

9

SLOTS AND FLAPS

General. In an effort to increase the ratio of *maximum speed* to *minimum speed,* modern conventional design has had to resort to various devices. Referring to the expression for minimum speed

$$V_{\min} = \sqrt{\frac{W/S}{C_{L\,\max}\,\rho/2}}$$

it is readily seen that for a given airplane design, based on a fixed wing, the landing speed is fixed by the specific dimension of the airfoil section. Closer inspection reveals that a reduced wing loading and higher maximum lift coefficient are the only possible variables and an increase of either usually means an increase of drag which, in turn, would reduce the top speed. That is to say, if the weight is fixed, an increase of wing area would evidently give a reduced wing loading and also a slower landing speed but the increased area would also add to the drag and reduce the maximum speed.

It would seem, then, that the criterion for airplane performance would be: as high a top speed as possible and as slow a landing speed as possible. The only remaining means to this end is to increase the maximum lift coefficient.

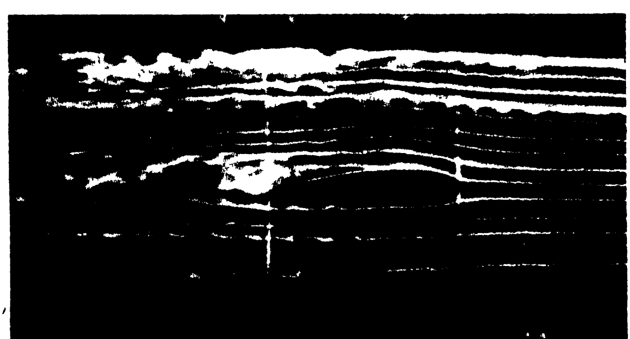
Slots. With the necessity for increased pay loads and desired high top speed, the high-powered supercharged engine came into general use. The development of greater horsepower had solved one-half the problem but there still remained the problem of slow, safe landing speeds.

One of the first devices to come into use was the slotted wing. The slot is a narrow opening just back of the leading edge and is made parallel to the span. Heretofore, the plain wing had been used and the maximum angle of attack was limited to 18° to 20° because the airflow was unable to follow the upper camber due to the great change in direction. However, the use of the slotted wing, where the leading edge of the main wing consisted of a small auxiliary airfoil, made it possible for the small airfoil in front to give a downward deflection to the air at high angles of attack, thus enabling the air to continue to flow smoothly over the upper camber.

Now with the aid of the slotted wing, not only was the maximum

Separation occurring on an airfoil at a high angle of attack.

Separation occurring on an airfoil at a low angle of attack.



Separation occurring on an airfoil at a low angle of attack but at an increased Reynolds Number.

Fig. 9-1

angle of attack increased to 28° to 30° but the maximum lift coefficient was increased about 50 percent.

Another great advantage in the slotted wings is that at high angles of attack where an ordinary wing would burble and become practically ineffective laterally, the slotted wing enables the air to flow smoothly over the ailerons. In this manner, the lateral control at high angles of attack and slow speeds is greatly improved.

At first, the *fixed slot* was used extensively but it was found that an improvement could be made if an *automatic slot* was installed. That is, at small angles of attack the drop in pressure on the upper camber tends to suck the air up through the fixed slot and to cause burbling which in turn increases the drag. The automatic slot is so constructed that at small angles of attack, where the dynamic pressure of the air is greatest at the nose or leading edge, the air pressure will push the auxiliary airfoil flush against the leading edge of the main wing. The mechanism of the automatic slot is made to move freely from a closed position at low angles of attack to an open position of one to two inches at high angles of attack. When the angle of attack is increased, the reduction in pressure on the upper camber tends to suck the auxiliary airfoil to its open position. Tests in the wind tunnel prove that the drag on the automatic slotted wing at small angles of attack is only slightly greater than the plain wing.

Flaps have also come into general use and have proved more popular with designers than the slotted wing. Flaps are, in appearance, like ailerons in that they are a hinged part of the trailing edge of the main wing. Flaps, however, extend across the span farther than ailerons and their deflection is always downward simultaneously. Like ailerons, the downward deflection of the flaps increases the upper camber of the wing and also forms a concave lower camber. The increase of upper camber and the dynamic pressure on the lower deflected surface causes an increase in lift coefficient. This increase in lift coefficient is generally about 50 percent greater than that for a plain wing of the same dimensions. The flap control, from the cockpit, is made so that, at small angles of attack (high speed), the flap can be pulled back into normal position and the drag will be the same as the plain wing. A combination of both slots and flaps has been used successfully on the slower type of commercial and military airplanes and in many cases the maximum lift coefficient was increased from 80 to 100 percent.

Zap Flap. Another form of flap is the split or *Zap* flap where only the lower portion of the trailing edge of the wing is deflected downward, leaving the upper camber fixed.

Fowler Flap. One of the more successful special devices is the

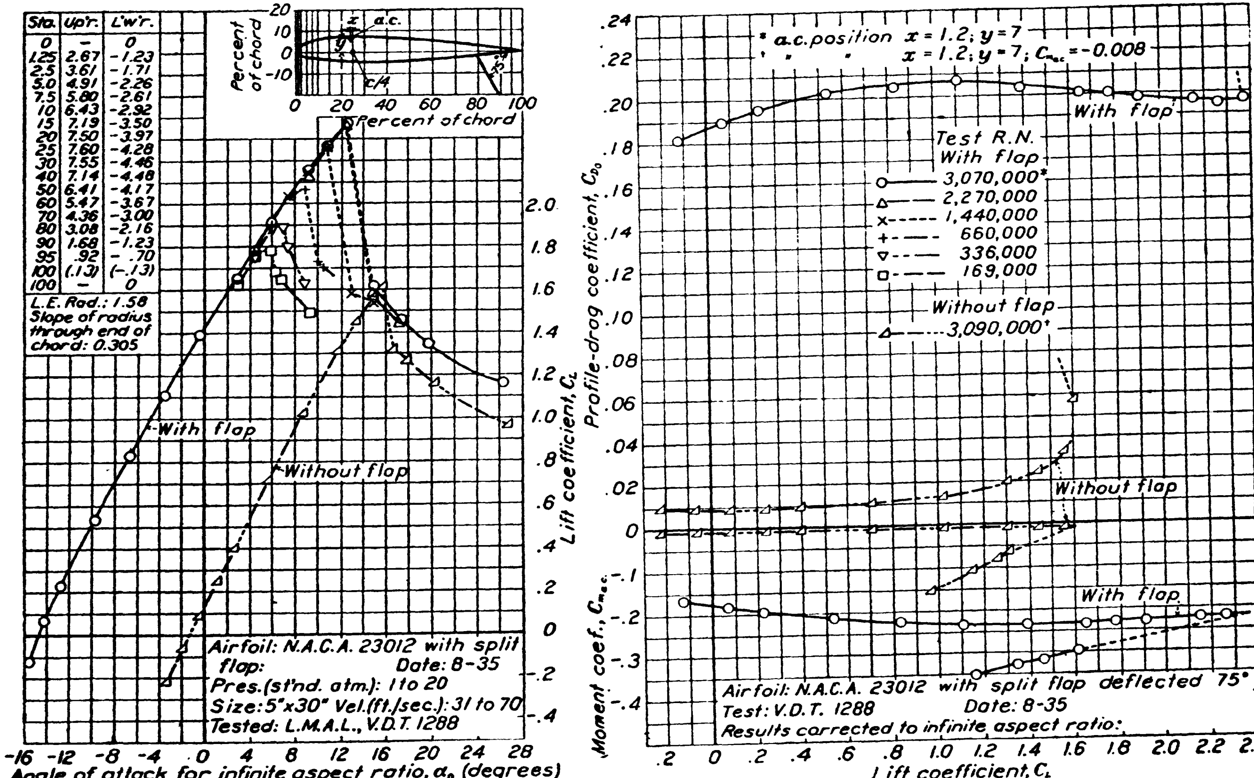


Fig. 9-2. N.A.C.A. 23012 with split flap deflected 75°.

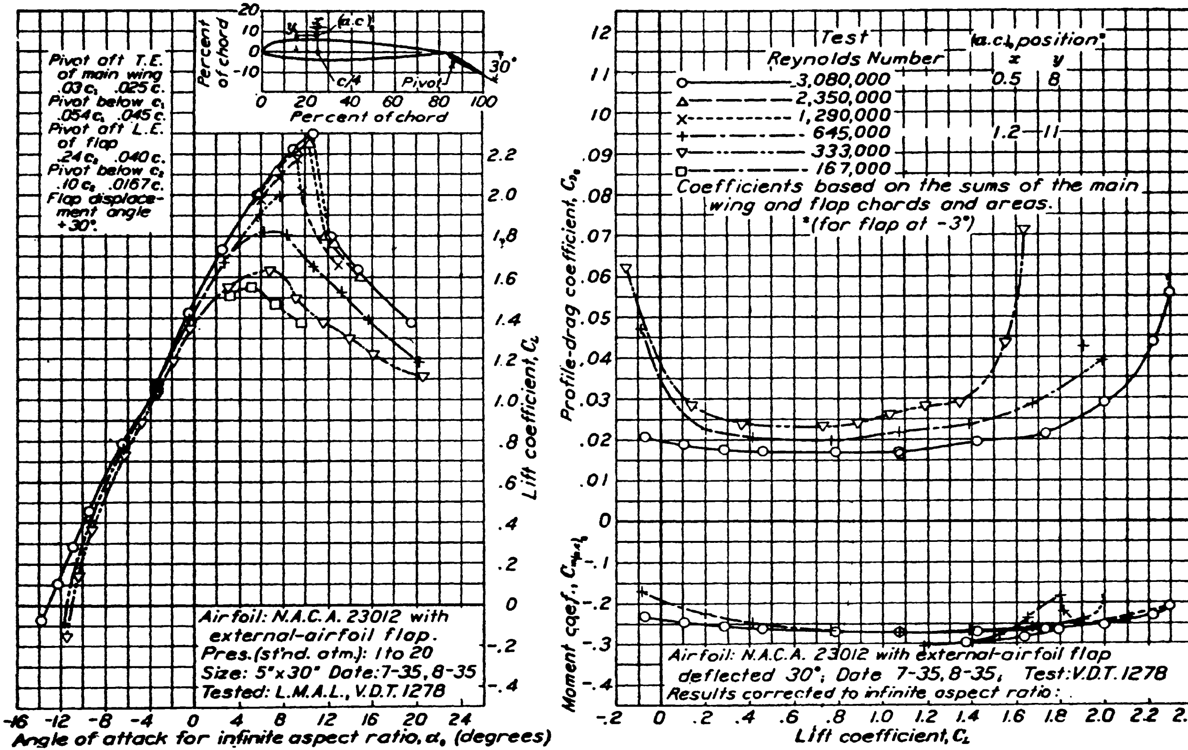


Fig. 9-3. N.A.C.A. 23012 with external airfoil flap deflected 30°.

Main wing section........	N.A.C.A. 23012	Main wing chord, c_1..............	0.833c	Datum chord, $c = c_1 + c_2$
Flap section............	N.A.C.A. 23012	Flap chord, $c_2 = 0.2c_1$...........	0.167c	

Fowler flap. It is made so that the flap, located on the lower camber trailing edge, may be deflected not only downward but also backward behind the trailing edge of the wing. In this way the wing area is actually increased and, of course, the maximum lift coefficient is increased. Thus the Fowler flap is a device which has solved both problem of decreased wing loading and increased lift coefficient.

10

PERFORMANCE

Definition. Performance of an airplane includes all the characteristics of an airplane in flight. The term performance is, nevertheless, used in a restricted sense in that only those flight qualities that are measurable quantitatively are included. The characteristics such as *stability, maneuverability,* etc., do not lend themselves readily to direct computation and are not included in performance as defined. Those performance characteristics that will be studied in this chapter are as follows:

(a) Maximum speed in level flight at sea level
(b) Minimum horizontal speeds at sea level
(c) Rate of climb at any altitude
(d) Speed and angle for maximum climb
(e) Maximum and minimum speed at altitude
(f) Rate of climb at altitude
(g) Absolute and service ceilings
(h) Endurance and range

Performance Calculations are necessary in the preliminary stages of the design to determine whether or not the original specifications are being met.

To cut down on the time and labor involved in making exact calculations, the following assumptions are usually made:

(a) That the motion in flight is *unaccelerated.*
(b) The *thrust* is equal to the *total drag* and acts in a horizontal direction.
(c) The lift on the wing is equal to the gross weight of the airplane.

These assumptions are justified because very little error is introduced in neglecting them.

Performance estimated for determining minimum speeds, maximum speeds, rate of climb, etc., may be obtained by a series of calculations involving horsepower required at various speeds and the horsepower available at these corresponding speeds. The data for determining power required to fly in horizontal flight may be found in the following ways:

(a) From wind-tunnel tests of the N.A.C.A.

(b) Wind-tunnel tests on the several components of the design (landing gear, cowling, etc.)
(c) Wind-tunnel test on the complete model
(d) Wind-tunnel test and flight tests of a similar design

It should be understood that these are only preliminary calculations, and the resulting accuracy is dependent upon the judgment of the engineer using the data. The exact performance data should and must be obtained from the actual flight tests.

Horsepower Required at Sea Level. In order to calculate the horsepower required at various speeds in level flight the following familiar formulas are used:

$$HP = \frac{(D_w + D_P)V}{375}$$

where HP = total power required
D_w = drag of wing in lb
D_P = drag of parasite and interference in lb
V = velocity in mph

The velocity in the above formula is expressed in miles per hour since it is customary to plot the values of horsepower required against corresponding airspeeds in miles per hour.

The formula for velocity in level unaccelerated flight is

$$V = \sqrt{\frac{W}{0.00256 \times S \times C_L}}$$

where V = mph
S = wing area in sq ft
W = weight of airplane in lb
C_L = lift coefficient

The usual method is to select various angles of attack ranging from angle of attack of zero lift to angle of attack for the maximum lift coefficient, *i.e.*, to select a series of angles of attack that will give corresponding lift coefficients in the flight range of the particular airfoil.

To find the wing drag at the corresponding velocity and angle of attack, found in the above expression, the following formula is used:

$$D_w = C_D \times 0.00256 \times S \times V^2 \quad \text{where } V = \text{mph}$$

The wing drag coefficients are usually available for an aspect ratio of 6, or for an infinite aspect ratio. If an aspect ratio other than that available is desired, corrections must be made as explained in the chapter on induced drag.

As has been discussed earlier the parasite-drag coefficients in the

cases of struts, wires, landing gears, fuselages, etc., can be obtained from various formulas and graphs from wind-tunnel tests.

The added drag due to interference of parts is even more difficult to calculate or appraise. If wind-tunnel data are not available, approximate calculations may be made by reference to known designs.

The formula for parasite drag, D_P, is

$$D_P = 0.00327 \times A_e \times V^2$$

where A_e = E.F.P.A.

V = velocity in mph

For more exact work, the above velocity should be the slipstream velocity of the airplane. For a cantilever type monoplane, all the parts which make up the parasite drag lie in the slipstream. The slipstream of the slower airplane is greater than the speed of flight and therefore the drag will be calculated more accurately if this value for slipstream velocity is used in the above expression. In the faster airplanes the forward velocity of the airplane is very nearly the same as the slipstream velocity and therefore, in all calculations for performance, the velocity of the airplane is considered the correct value to use for practical purposes.

These calculations can best be shown by the following example:

Problem I. Plot HP required versus airspeed for a monoplane weighing 3000 lb and having a N.A.C.A. 23009 rectangular wing of 42-ft span and 7-ft chord. The parasite drag has an E.F.P.A. of 4.3 sq ft.

TABLE IV

1 α	2 C_L	3 C_D	4 V (mph)	5 D_w	6 D_P	7 HP
0	0.08	0.009	222	334	692	607
1	0.15	0.01	162.3	198.2	370	246
2	0.23	0.012	131	154.9	246	139
3	0.30	0.014	114.7	138.4	184.8	98.6
4	0.38	0.017	102	132.9	146.2	75.8
8	0.69	0.035	75.7	150.7	80.4	46.7
12	0.97	0.065	63.8	199.3	57.3	43.6
16	1.28	0.11	55.7	256.3	43.6	44.4
18	1.40	0.135	53.2	287.5	39.8	46.4
20	1.20	0.16	57.4	396.2	46.3	67.6
22	1.10	0.29	60.0	786.0	50.6	134

Column 1.—In this column the angles of attack of the wing are tabulated for reference. The angular range should be started just above the angle of zero lift and continue, in intervals as shown, to a few angles beyond that for maximum lift.

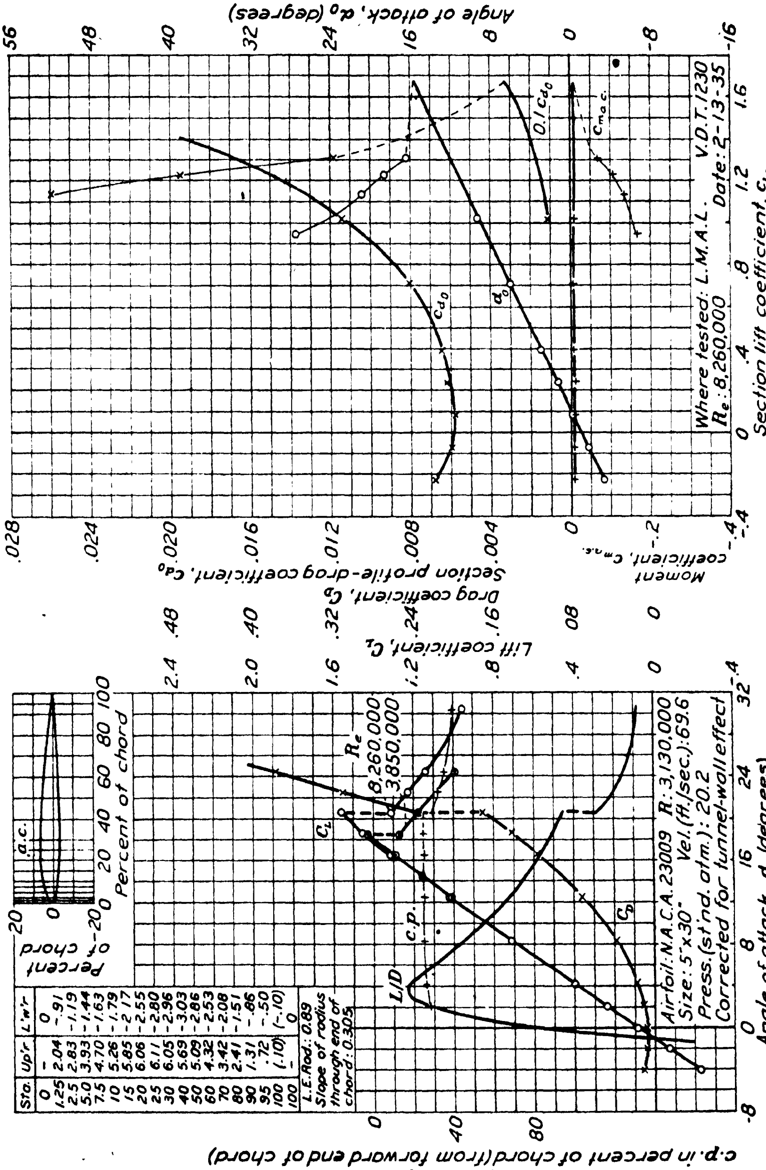


Fig. 10-1. N.A.C.A. 23009 airfoil.

Columns 2,3.—The values of C_L and C_D may be obtained from Fig. 10-1 corresponding to the angle of attack listed in Column 1.

Column 4.—As previously explained, the velocity in miles per hour may be calculated by the expression

$$V = \sqrt{\frac{W}{0.00256 \times S \times C_L}}$$

For any particular design, the quantity $\sqrt{\frac{W/S}{0.00256}}$ will remain constant for the series of calculations. Therefore, for this particular problem the expression for velocity may be simplified thus:

$$V = \frac{63.1}{\sqrt{C_L}}$$

That is, values for Column 4 were obtained by dividing the square root of Column 2 into 63.1.

Column 5.—Here the drag of the wing is calculated by the formula:

$$D_w = C_D \times 0.00256 \times S \times V^2$$

This expression likewise may be simplified to:

$$D_w = 0.75264 \times C_D \times V^2$$

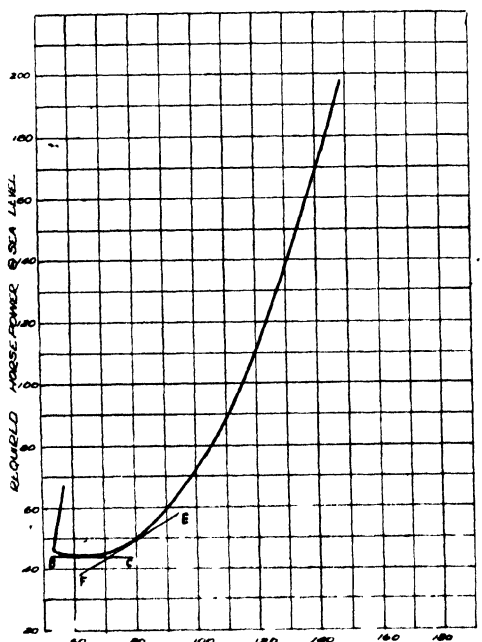


Fig. 10-2.

Once simplified, Column 5 may be obtained by multiplying 0.75264 by Column 3 and then multiplying by the square of the velocity in Column 4.

Column 6.—The parasite drag is found by the formula:

$$D_P = 0.00327 \times A_e \times V^2$$

Here again, the expression is simplified for the particular problem and since the E.F.P.A. is given as 4.3 sq ft the expression for parasite drag may be simplified thus:

$$D_P = 0.014061 \times V^2$$

Column 7.—The total HP required is found by the formula:

$$HP = \frac{(D_w + D_P)V}{375}$$

That is, total HP required is found by adding Columns 5 and 6 and multiplying the sum by Column 4 and dividing the product by 375.

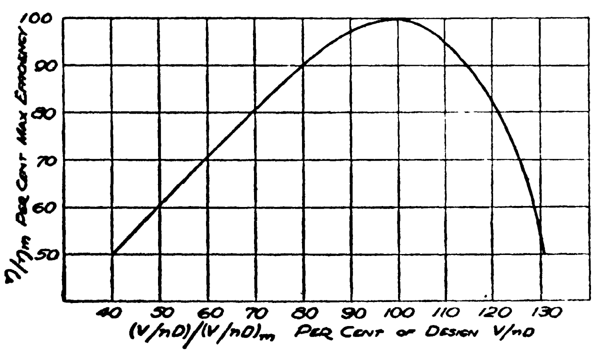


Fig. 10-3.

The results of the computations tabulated in Table IV and Column 7 (total HP required) are plotted against corresponding values in Column 4 (airspeed in mph) in Fig. 10-2.

General Performance Curves. It has been pointed out in the N.A.C.A. Report No. 168 that, if the percent of maximum efficiency (η/η_m) is plotted against the corresponding percent of

$$\frac{V}{nD}\left(\left(\frac{V}{nD}\right)\Big/\left(\frac{V}{nD}\right)_m\right)$$

for wooden propellers of various pitch ratios, the resulting curves for all pitches would very nearly coincide. In this plot, Fig. 10-3, η_m is the peak efficiency for each pitch ratio and $(V/nD)_m$ is the V/nD at that peak. For a metal propeller the curve is somewhat different, but a similar relationship exists.

To find the full-throttle power at speeds less than maximum speed, say for climbing, consider that, for a fixed-pitch propeller, the blade elements are working at a higher angle of attack. At the higher angle of attack the drag on the blades is greater and the engine will necessarily be held down to less rpm. At this slower engine speed the horsepower delivered will be less. Also when the blade element works at an angle of attack beyond that for best L/D, the efficiency will drop off. The drop in engine speed with reduced airspeed is shown for typical *peak-efficiency* propellers in Fig. 10-4. For typical

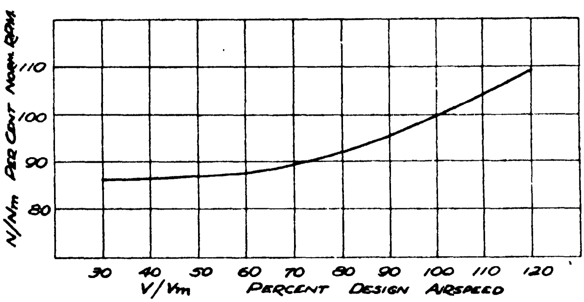


Fig. 10-4.

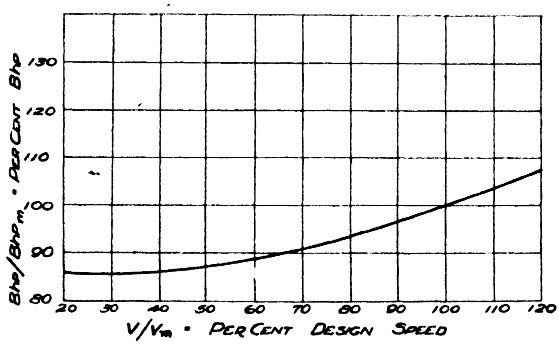


Fig. 10-5.

peak-efficiency propellers the drop in brake horsepower (B. HP) with airspeed is plotted in Fig. 10-5.

Horsepower Available is always the horsepower developed at the engine crankshaft multiplied by the propeller efficiency. In the graphs above, it is seen that neither the horsepower developed nor the propeller efficiency is constant with change of airspeed, *i.e.*, when the airplane is climbing, the horsepower developed is reduced. To calculate the horsepower available for a fixed-pitch propeller at different airspeeds, it is necessary to calculate the power-speed coefficient and the corresponding efficiency. The method of determining the power

available can best be shown by the calculations in the following example.

Problem II. The same airplane in Problem I has an engine rated at 220 HP at 1800 rpm. It is desired to find and plot the data of HP available against velocity at sea level.

Solution: The propeller efficiency (η) is assumed to be 82 percent. Then the HP available, for the first approximation, is $220 \times 0.82 = 180$ HP. Then from the plot of HP required (Fig. 10-2), 180 HP is required at 144 mph. Knowing the above design speed (144 mph), the rated HP and engine rpm, the C_S may be found.

$$C_S = \frac{0.638 \times mph}{HP^{1/5} \times rpm^{2/5}}$$

$$= \frac{0.638 \times 144}{220^{1/5} \times 1800^{2/5}}$$

$$= 1.55$$

Then from Fig. 8-7 a propeller is selected with blade angle of 19° (by interpolation) with V/nD of 0.77 (η = 83 percent)

Then the diameter is

$$D = \frac{V}{N \times \frac{V}{nD}}$$

$$= \frac{144 \times 88}{1800 \times 0.77}$$

$$= 9.14 \text{ ft}$$

TABLE V. SEA-LEVEL POWER AVAILABLE FOR FIXED-PITCH PROPELLER

1	2	3	4	5	6	7	8	9	10
V (Mph)	% Design V	% Normal Rpm	Rpm	% Rated HP	B.HP	C_S	β	η	HP Available
50	34.7	87.0	1568	87	191.5	0.587	19°	0.55	105.3
60	41.7	87.2	1570	87.2	191.9	0.703	19°	0.62	119.3
70	48.7	87.5	1575	87.9	193.5	0.820	19°	0.675	129.4
80	56.6	88.0	1585	88.3	194.0	0.934	19°	0.72	140.0
90	62.5	88.5	1594	89.1	196	1.048	19°	0.755	148.1
100	69.4	90.0	1620	90.7	199.5	1.154	19°	0.77	153.8
110	76.4	91.5	1649	92.0	202.2	1.250	19°	0.80	161.9
120	83.3	94.0	1693	94.0	207	1.350	19°	0.81	167.7
130	90.3	97.0	1748	96.1	211.8	1.450	19°	0.82	173.5
140	97.2	99.0	1783	98.8	217.5	1.520	19°	0.83	180.0
144	100.0	100.0	1800	100.0	220	1.55	19°	0.830	182.5

Column 1.—The airspeeds are chosen from the approximate stalling speed to the maximum speed in ten-mile intervals.

Column 2.—In this column are the items in Column 1 divided by the design speed (144 mph).

Column 3.—The percent of normal rpm is found for the corresponding item in Column 2 by use of Fig. 10-4.

Column 4.—The rpm of the engine are calculated by multiplying the percent of normal rpm in Column 3 by the normal rated rpm of the engine (1800 rpm).

Column 5.—The percent of rated HP in this column is found for the corresponding percent of design speed in 2 by the aid of Fig. 10-5.

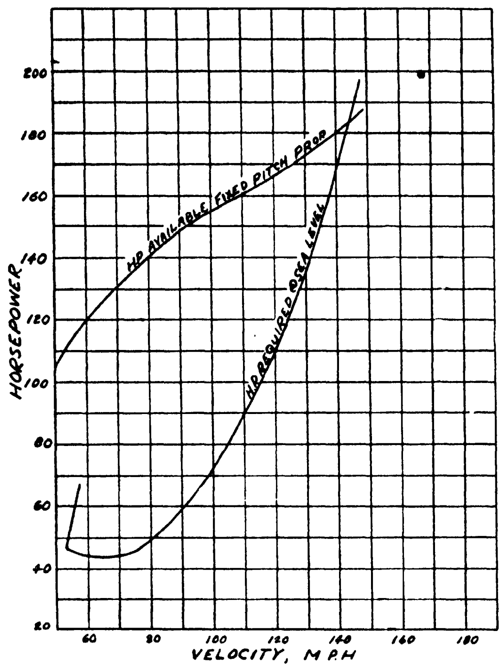


Fig. 10-6.

Column 6.—In this column the B.HP at each airspeed is calculated by multiplying the percent of rated HP in Column 5 by the rated HP of the engine (220 HP).

Column 7.—The power-speed coefficient is found by substituting the velocity in Column 1, the rpm in Column 4, and the B.HP in Column 6, in the expression

$$\text{power-speed coefficient} = C_S = \frac{0.638 \times mph}{HP^{1/5} \times rpm^{2/5}}$$

The expression for C_S is for sea-level conditions.

Column 8.—Since the propeller in this particular example has a fixed pitch, the blade angle β will be constant at all speeds. The value of $\beta = 19°$ was found by computations as suggested in the chapter on propellers.

Column 9.—The values for efficiency, η, are found for the corresponding values of the C_S in Column 7 with the aid of Fig. 8-7.

Column 10.—The values for HP available are found by the product of the corresponding items in Columns 6 and 9.

The next step is to plot the horsepower available in Column 10 against the corresponding airspeed in Column 1. This plot of power-available curve should be made on the same graph as the power-required curve (see Fig. 10-6). With these data before him, the designer is ready to determine the performance characteristics of the airplane.

In the case of an airplane equipped with a constant-speed propeller, the rpm will remain the same regardless of the forward speed of flight. The constant-speed propeller has advantages over the fixed-pitch propeller but because of the added weight, it may or may not be suitable for the particular design. In either case, the horsepower-required curve for one particular airplane will remain the same but the horsepower-available curve will be different, more horsepower being available with the use of the constant-speed propeller. The horsepower available may be found easily by calculating the value of V/nD and the C_s, then, by use of propeller-selection charts, the blade angle and efficiency may be determined. Most constant-speed propellers have an angular range limited to 10 degrees and if the blade angles determined, as explained above, fall outside this range, the calculations will necessarily be the same as for the fixed-pitch propeller.

To show the advantages of the constant-speed propeller over the fixed-pitch propeller more clearly, the same airplane, as in the first example, will be used. The following table gives results of the computations.

Problem III. Given: The same airplane in Problem I with an engine rated at 220 HP at 1800 rpm. The design speed is 144 mph and propeller diameter is 9.14 ft as determined before by assuming an efficiency of 82 percent. If this airplane is equipped with a constant-speed metal propeller, calculate and plot HP available.

TABLE VI. POWER AVAILABLE FOR CONSTANT-SPEED PROPELLER

1	2	3	4	5	6	7	8
V	B.HP	Rpm	C_S	V/nD	β	η	HP Available
50	220	1800	0.538	0.2675	14°	57.5	126.5
60	220	1800	0.646	0.321	14°	63.2	139.0
70	220	1800	0.754	0.3743	14.5°	68.0	149.8
80	220	1800	0.862	0.428	15°	72.0	158.5
90	220	1800	0.969	0.482	15°	75.5	166.2
100	220	1800	1.077	0.535	16.5°	77.0	169.5
110	220	1800	1.184	0.5885	17°	79.0	174.0
120	220	1800	1.292	0.642	17.2°	80.5	177.2
130	220	1800	1.399	0.695	17.5°	81.5	179.4
140	220	1800	1.508	0.749	18°	82.5	181.7
144	220	1800	1.55	0.77	19°	83.0	182.5

Column 1.—Velocities are selected from approximate stalling speed to maximum speed at intervals of 10 mph.

Columns 2, 3.—Since a constant-speed metal propeller is used on a direct-drive crankshaft, the rated HP and rpm in Columns 2 and 3 respectively will be constant throughout the flight range.

Column 4.—The power-speed coefficient, for sea-level conditions, is calculated by substituting the corresponding values of velocity, HP and rpm in the formula,

$$C_S = \frac{0.638 \times mph}{HP^{1/5} \times rpm^{2/5}}$$

This series of calculations may be greatly simplified because, for each particular problem, there will be certain values that will remain constant throughout the calculations. For this example, the expression will be

$$C_S = \left(\frac{0.638}{220^{1/5} \times 1800^{2/5}}\right) \times mph$$

The quantity in parenthesis is the constant and may be simplified still further

$$C_S = \left(\frac{0.638}{2.945 \times 20.1}\right) \times mph$$

$$= 0.01077 \times mph$$

With the above expression it is only necessary to multiply 0.01077 by the corresponding velocities listed in Column 1.

Column 5.—In this column the values of the V/nD, for the corresponding velocities in Column 1, may easily be calculated since the rpm and the diameter are fixed.

$$\frac{V}{nD} = \frac{mph \times \frac{88}{60}}{\left(\frac{rpm}{60} \times D\right)}$$

Simplified still further

$$\frac{V}{nD} = \frac{mph \times 88}{rpm \times D}$$

For this particular problem

$$\frac{V}{nD} = \frac{mph \times 88}{1800 \times 9.14}$$

$$= mph \times 0.00535$$

Column 6.—Using the calculated values of C_S and V/nD, the blade

angle β may be determined by the use of the lower curves in Fig. 8-7.

Column 7.—The efficiency for the corresponding C_S and the blade angle β is determined by the use of the upper curves in Fig. 8-7.

Column 8.—The HP available for the airspeeds in Column 1 may be determined by multiplying the rated HP (220) by the corresponding efficiencies in Column 7.

Plotting the horsepower-available curve for the constant-speed propeller on the same graph as the one for the fixed-pitch propeller, it

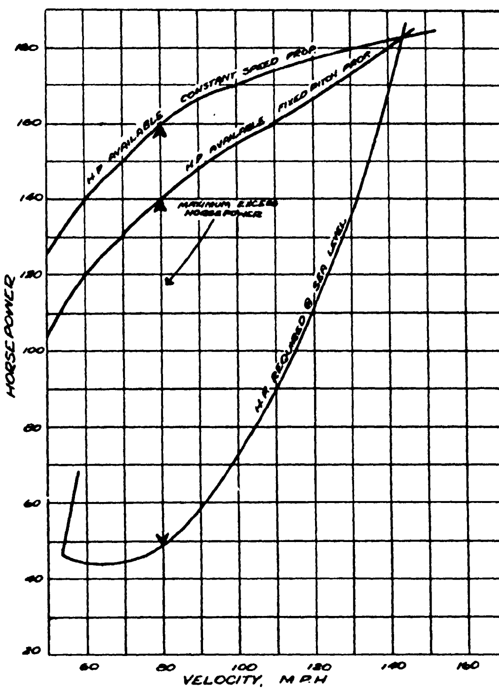


Fig. 10-7.

is at once evident that the excess horsepower is greater for the constant-speed propeller. See Fig. 10-7. This excess horsepower, as will be explained later, has a direct relation to the performance, *i.e.*, the constant-speed propeller curves producing greater excess power means improved performance characteristics.

Determination of Maximum Speed. The conditions for *maximum speed* are that the power required for level flight be equal to the power available at the propeller. The maximum speed for level flight may be read from the graph at the intersection of the horse-

power-required and horsepower-available curves. From Fig. 10-6 it is seen that the maximum speed is 145 mph for the fixed-pitch propeller and for velocities less than the maximum, there is more than necessary horsepower available. This means, at that speed, the engine should be throttled down. For speeds greater than the maximum, the ordinate of the horsepower-required curve is greater than that of the horsepower-available curve and this means, of course, that the airplane cannot fly at that speed because there is not sufficient power available.

Minimum Speed or stalling speed of the airplane may easily be determined by the formula

$$V = \sqrt{\frac{W}{\rho/2\, S\, C_{L\ \max}}}$$

For the airplane referred to above, the minimum or stalling speed was calculated to be 53.2 mph. The landing speed of an airplane is sometimes estimated to be a few miles per hour less than the actual calculated stalling speed. The reason for this is that, as the airplane approaches the ground, there is a cushioning effect due to air being compressed between the wing and the ground, the phenomenon being the equivalent of an increase in maximum lift coefficient. Many elaborate tests and calculations have been made to determine the magnitude of this effect and, in the N.A.C.A. Technical Note No. 349, the variations between actual and theoretical landing speeds were found to be of the order of 1 percent or 2 percent.

Rate of Climb. The maximum *rate of climb* may be found easily by an examination of the curves of horsepower-required and horsepower-available. The expression for rate of climb (R.C.) is:

$$\text{R.C.} = \frac{33{,}000 \times \text{excess HP}}{\text{weight of plane (lb)}}$$

Since one horsepower is 33,000 ft-lb per minute, if this is multiplied by the amount of *excess horsepower* and divided by the weight of the airplane in lb, the rate of climb will be in fpm.

$$\text{R.C.} = \frac{\text{ft-lb per minute}}{\text{lb}} = \text{fpm}$$

For the airplane in Fig. 10-6 equipped with the fixed-pitch propeller, the maximum difference at the same airspeed between the ordinates for horsepower required and horsepower available is found to be 91 HP. Then the maximum rate of climb for the airplane in Fig. 10-6, when the gross weight is 3000 lb, is

$$\text{R.C.} = \frac{33{,}000 \times 91}{3000}$$

$$\text{R.C.} = 1000.3 \text{ fpm}$$

For the same airplane, equipped with the constant-speed propeller, it is seen from Fig. 10-7 that the ordinate for the horsepower-available curve is greater than that for the fixed-pitch propeller. This means that the excess horsepower is greater for the constant-speed propeller and therefore the rate of climb will be increased. From Fig. 10-7, it is seen that, at the same airspeed, the maximum difference in the ordinate for horsepower available and horsepower required is 110 HP producing a maximum rate of climb of

$$R.C. = \frac{33{,}000 \times 110}{3000}$$

$$= 1210 \text{ fpm}$$

Speed of Best Climb. When the maximum rate of climb is determined as explained above, the airspeed for that *best* rate of climb is found by denoting the velocity at which point the maximum excess horsepower is located. Examining the curves of horsepower-available and -required versus velocity, Fig. 10-7, it can be seen at once that the speed of best climb corresponding to the position for maximum excess horsepower is 80 mph: For this particular airplane, the pilot will get the maximum rate of climb if he climbs the airplane at the indicated airspeed of 80 mph.

For some particular flight problems the speed for *steepest climb* rather than the speed for best climb may be desired. In order to clear objects on a take-off, if the angle of attack is increased even farther than that for best climb, the airspeed will decrease but the ratio of climb to forward speed may increase. To find the angle of

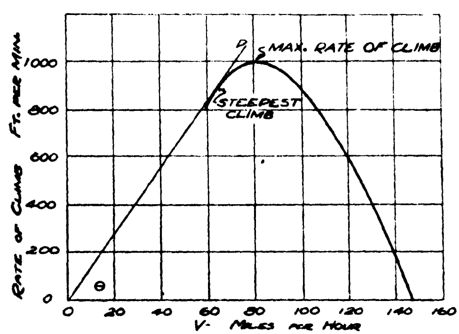


Fig. 10-8.

climb for airspeed for steepest climb for various speeds it is only necessary to plot them as shown in Fig. 10-8. The peak of this curve is called the *maximum rate of climb* at sea level. However, if a line, *OP*, is drawn through the origin tangent to this curve, the

point of tangency will be the speed for steepest climb and the angle of climb will be

$$\theta = \tan^{-1} \frac{875 \text{ ft/min}}{65 \text{ mph}} = \frac{9.95 \text{ mph}}{65 \text{ mph}}$$
$$= 0.153$$
$$= 8^\circ\ 42' \text{ (approx)}$$

The airspeed for steepest climb determined by the point of tangency is 65 mph. In other words, this particular airplane should be flown at an airspeed of 65 mph to attain the greatest possible height in the shortest distance.

Power Required at Altitude. Up to this time, the only performance considered has been that at sea level. Although performance calculations at sea level are important, the designer must also determine the characteristics of his airplane at the altitude where it will actually perform.

It was shown earlier that, at any altitude above sea level when the plane was flying level at constant angle of attack, the true airspeed increased due to the decrease in air density. The true airspeed at altitude is equal to the airspeed at sea level multiplied by the reciprocal of the square root of the density ratio.

$$V = V_0\sqrt{\frac{\rho_0}{\rho}}$$
$$= V_0\sqrt{\frac{1}{\sigma}}$$

where ρ_0 = density at sea level
ρ = density at altitude
$\rho/\rho_0 = \sigma$ (sigma) = density ratio (see Table I)

Likewise, it has been proved that because airspeed at altitude increases in the same ratio that the density decreases, the drag at altitude is the same as at sea level for a constant angle of attack. Therefore, the power required at altitude is

$$HP = \frac{D_0 \times V_0}{375} \times \sqrt{\frac{1}{\sigma}}$$
$$= HP_0\sqrt{\frac{1}{\sigma}}$$

Then in order to calculate speed and power required at altitude it is only necessary to know the values of speed and power required at sea level and multiply them by the square root of the density ratio for that altitude.

Problem III. Using the data for HP required at sea level for the airplane in Problem I, calculate the speed of flight and power re-

quired for level flight for the same angles of attack at standard altitudes of 5000 ft and 10,000 ft.

Solution: Sample Calculations. For 5000 ft standard altitude, read $\sqrt{1/\sigma} = 1.077$ (Table I). For the angle of attack corresponding to the velocity at sea level of 57.4 mph, calculate the velocity at 5000 ft.

$$V = 57.4 \times 1.077 = 61.8 \; mph$$

Since at this speed the HP required is 67.6, the HP required at 5000 ft is

$$HP = 67.6 \times 1.077 = 72.8$$

TABLE VII. SPEED AND HORSEPOWER REQUIRED AT 5000 FT

Velocity (Mph), Sea Level	HP Required, Sea Level	Velocity (Mph), 5000 Ft	HP Required, 5000 Ft
220	607	241	654
162.3	246	174.8	264.3
131	139	141	149.7
114.7	98.6	123.4	106.2
102	75.8	109.9	81.6
75.7	46.7	81.5	50.25
63.8	43.6	68.6	46.9
55.7	44.4	59.9	47.8
53.2	46.4	57.3	49.9
57.4	67.6	61.8	72.7
60	134	64.6	143

Sample Calculations. For 10,000 ft, read $\sqrt{1/\sigma} = 1.164$. For the angle of attack corresponding to the velocity at sea level of 57.4 mph, calculate the velocity at 10,000 ft.

$$V = 57.4 \times 1.164 = 66.9 \; mph.$$

Since at sea level, at this speed, power required is 67.6, then the power required at 10,000 ft is

$$HP = 67.6 \times 1.164 = 78.7$$

TABLE VIII. SPEED AND HORSEPOWER REQUIRED AT 10,000 FT

Velocity (Mph), Sea Level	HP Required, Sea Level	Velocity (Mph), 10,000 Ft	HP Required, 10,000 Ft
222	607	258.2	706
162.3	246	189	286.5
131	139	152.4	161.9
114.7	98.6	133.5	114.8
102	75.8	118.8	88.2
75.7	46.7	88.1	54.3
63.8	43.6	73.4	50.7
55.7	44.4	64.8	51.7
53.2	46.4	61.8	54
57.4	67.6	66.8	78.7
60	134	69.8	156

The above calculated airspeeds and power required at altitudes of 5000 ft and 10,000 ft are plotted on the same graph with the sea-level power in Fig. 10-9. It may be noted that the effect of airplane operation at altitudes is to move the curve of horsepower required to the right and upward from the original position of the sea-level power-required curve. It is interesting to note that at a given airspeed an increase in altitude makes the power required greater at low

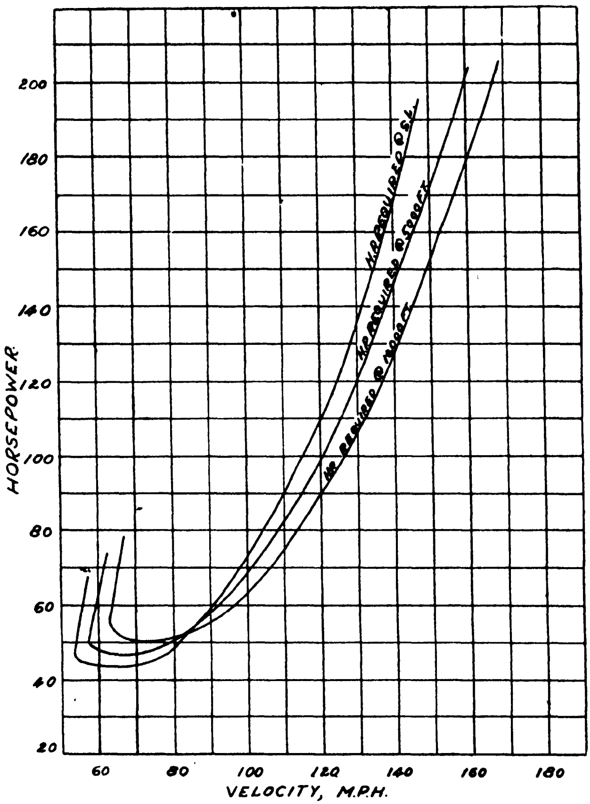
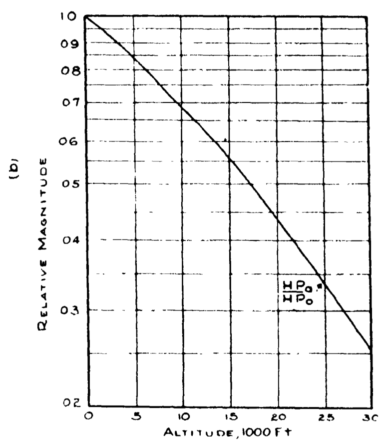


Fig. 10-9.

speeds (large angle of attack) and less at high speed (small angle of attack). It follows then, if the power available from the engine did not drop off at altitude, the maximum speed of the airplane would be greater at higher altitudes. This particular advantage is realized if the engine is equipped with a supercharger. For most engine-propeller units on lighter airplanes, however, there is no altitude engine and the power falls off to such an extent that the top speed at altitude is usually less than that at sea level.

Horsepower Available at Altitude. For an airplane engine at a given rpm the indicated horsepower drops off in practically the same ratio as the density of the air. This is because the percentage of oxygen content is practically independent of altitude. It is a fact that the propeller offers less resistance to rotation because of the decrease in density and it would seem that the propeller would turn over faster and develop more horsepower. This, however, is not the case since the engine power drops off faster than the density and the net result of an increase in altitude at a given airspeed is a drop in propeller rpm. That is, a reduction in rpm means a decrease in power output and also with the decrease in air density, even at the same rpm, the engine will develop less power. This would necessarily result in a change in V/nD for the propeller and, for a fixed-pitch



Variation of Full Throttle Horsepower With Altitude at Constant Air Speed

Fig. 10-10.

propeller, this change would also mean a decrease in propeller efficiency. In the N.A.C.A. Report No. 408, the net result of the changes in B.HP, rpm, and propeller efficiency with altitude, for engines with typical mechanical efficiencies, were plotted as a graph of HP/HP_0 against altitude for constant airspeed (Fig. 10-10).

Then, with the aid of Fig. 10-10 the plot of horsepower available at altitude against speed can be made when the corresponding values of sea-level power-available are known. This procedure can be illustrated best by the following example.

Problem V. Given: The same airplane in Problem II and the corresponding values of speed and HP available at sea level. Using the above values, calculate the HP available at each airspeed at standard altitudes of 5000 ft and 10,000 ft.

Solution: From Fig. 10-10 read for 5000-ft altitude, $HP/HP_0 = 0.83$. Next, tabulate the known values of speed and power available at sea level (Problem II) and multiply each value of power available at sea level by 0.83 to get the power available at 5000 ft.

Sample Calculation. For velocity of 50 mph the HP available at sea level is 105.3, then the HP available at 5000 ft is 105.3 × 0.83 = 87.4 HP.

TABLE IX. HORSEPOWER AVAILABLE AT 5000 FT

V_0	HP_0	HP
50	105.3	87.4
60	119.3	99.1
70	129.4	107.4
80	140.0	116.2
90	148.1	123.0
100	153.8	127.6
110	159.9	132.8
120	165.9	137.8
130	172.5	143.2
140	179.5	149.0
144	182.5	151.5

For calculation of HP available at 10,000 ft, read from Fig. 10-10 for 10,000 ft altitude $HP/HP_0 = 0.68$. Tabulate the known values of speed and power-available at sea level and multiply sea-level power-available by 0.68 to get power available at 10,000 ft.

Sample Calculation. HP available at 10,000-ft altitude at 50 mph = 105.3 HP × 0.68 = 71.6 HP.

TABLE X. HORSEPOWER AVAILABLE AT 10,000 FT

V_0	HP_0	HP
50	105.3	71.6
60	119.3	81.2
70	129.4	88.0
80	140.0	95.2
90	148.1	100.8
100	153.8	104.6
110	159.9	108.8
120	165.9	112.9
130	172.5	117.4
140	179.5	122.1
144	182.5	124.2

The above calculated values for horsepower available at 5000 ft and 10,000 ft are then plotted on the same graph as the horsepower required in Fig. 10-11 and with these data before him the designer may easily determine the performance characteristics at altitude.

Maximum and Minimum Speeds at Altitude are easily determined by noting the point of intersection of the available and required curves for the respective altitude. For example, in Fig. 10-11 the

power-required and power-available curves for 5000 ft intersect to give a maximum speed of 141 mph. In the same figure the curves for 10,000 ft intersect to give a maximum speed of 136.5 mph. If the altitude were increased even more, then the intersection of the curves would occur at a still lower maximum speed. As the altitude increases, the horsepower-required curve will move upward and to the right and at the same time the available curves will move

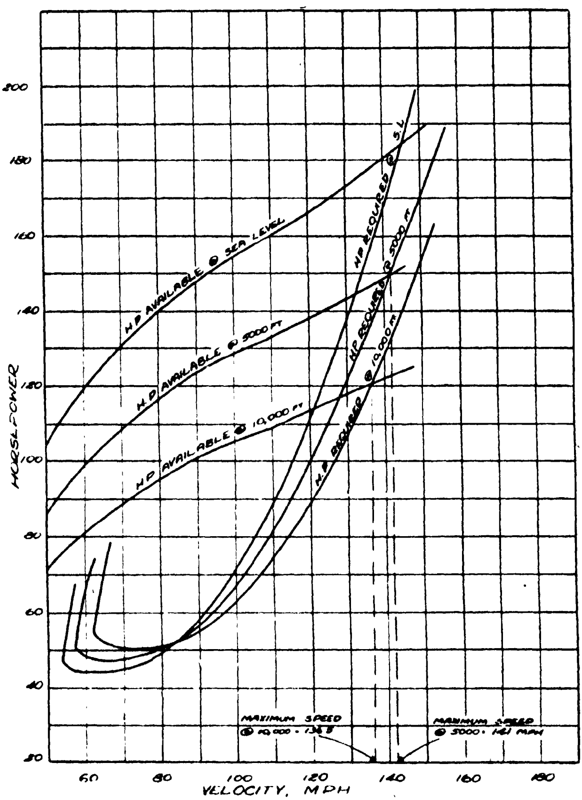


Fig. 10-11.

downward. At some greater altitude, then, the power-required curve will occur above the power-available curve and will not intersect at any point. This means that, at this particular altitude, the airplane is unable to operate because the power available is insufficient.

The minimum speed at altitude is usually the lower limit of the horsepower-required curve. In other words, from Fig. 10-11 the minimum speed at 5000 ft is seen to be 57.3 mph, and the minimum

speed at 10,000 ft is 61.8 mph. Usually, at some higher altitudes, the stalling speed of the airplane will be higher than the calculated stalling speed because the power available will intersect the power-required curve. That means, in some cases, the intersection of the two curves occurs not only to determine maximum speed but also to determine stalling speed. In a case of this kind, range of flight operation is limited to the speeds between the lower intersection (minimum speed) and the upper intersection (maximum speed).

Rate of Climb at Altitude is determined in the same manner as for sea level. For altitudes above sea level, the power-required curves move up and toward the right and the power-available curves move downward. That is, at altitude, the power required is greater and

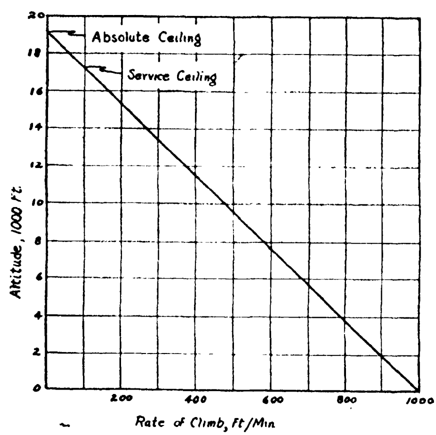


Fig. 10-12.

the power available less. The maximum difference between these curves at the same speed will determine the maximum rate of climb at the particular altitude for which the calculations are made. From Fig. 10-11 the maximum excess power for 5000 ft is 67 HP at 85 mph. The maximum rate of climb at 5000 ft is

$$\frac{67 \times 33{,}000}{3000} = 737.5 \text{ ft/min}$$

In the same manner, the maximum excess power for 10,000 ft is 44.5 HP at 90 mph, giving a maximum rate of climb of

$$\frac{44.5 \times 33{,}000}{3000} = 489 \text{ ft/min}$$

The airspeed for maximum climb, as shown, is greater with an increase in altitude and is closely approaching the airspeed for least power required. In other words, the angle of attack for this best climb is

practically the same as the angle of maximum $C_L^{3/2}/C_{D\,\text{total}}$ of the complete airplane. The rate of climb curve, when plotted against altitude, is approximately a straight line for unsupercharged engines (Fig. 10-12). It is also a straight line for supercharged engines if the climb is made with a constant throttle setting. However, if the climb is made at constant manifold pressure to the critical altitude of the supercharger, the rate of climb will be a curve below the critical altitude and a straight line above the critical altitude.

Ceiling. *Absolute ceiling* is the limiting altitude of the airplane, that is, there remains no excess power available for climb and the rate of climb becomes zero. *Service ceiling* is another term used to express the performance of an airplane and is defined as the altitude at which the rate of climb is 100 ft/min. In order to obtain the absolute and service ceilings of an airplane it is only necessary to plot the rates of climb for two different altitudes as shown in Fig. 10-12. The calculated rates of climb for the airplane in Problem I have been plotted against altitude in Fig. 10-12 and the resulting curve is practically a straight line. This straight line drawn through these points intersects the zero ordinate to determine the absolute ceiling of this airplane to be 19,200 ft. The service ceiling is noted to be 17,400 ft.

Endurance. To attain the maximum possible *endurance* for a particular airplane, it is necessary that the pilot fly at the airspeed for minimum power required. This means that at the velocity for minimum power required will be the speed at which the least fuel consumption in gallons per hour will be obtained. If the airplane is to make an endurance record for time in the air, the gasoline supply should be consumed as economically as possible. Referring to an earlier explanation, the airplane should be flown at an angle of attack corresponding to the minimum value of $C_D/C_L^{3/2}$ for the least horsepower required. However, the angle of attack must be for the corresponding minimum $C_{D\text{total}}/C_L^{3/2}$ which means the drag of the wing plus the parasite drag must be considered. If the total horsepower-required curve has been plotted the minimum power required is easily found.

Example: Using Fig. 10-2 for the airplane in Problem I, it is desired to find the angle of attack and velocity for minimum $C_{D\,\text{total}}/C_L^{3/2}$.

Solution: Draw a line, BC, horizontal and tangent to the HP-required curve Fig. 10-2; the point of tangency being the point of velocity for minimum power required. The point of tangency, for this particular airplane, determines the minimum velocity to be 63.8 mph and the minimum HP required is 43.6. The corresponding angle of attack for minimum $C_{D\,\text{total}}/C_L^{3/2}$ from Table IV is 12°.

It follows, then, for the airplane in the above problem, the pilot must fly level at an airspeed of 63.8 mph to obtain the greatest possible endurance flight. This will be true only when the airplane weighs 3000 lb. As time elapses, the airplane will become lighter due to the fuel consumption and, although he must fly at the angle of attack of minimum $C_{D\,\text{total}}/C_L^{3/2}$, the decrease in weight will decrease the necessary velocity. Decreasing the weight, the power required is less hence the throttle setting can be reduced. This decrease in throttle setting means less fuel consumption in gallons per hour. For an airplane to attain its absolute maximum endurance, the pilot should have a table of best airspeeds to fly at various stages of the flight.

In the N.A.C.A. Technical Report No. 234, the following expression, the *Brequet Endurance Formula*, is given:

$$\text{Endurance (hours)} = 750\,\frac{\sqrt{W}}{V}\left(\frac{L}{D}\right)\frac{\eta}{C}\left[\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right]$$

$$\left.\begin{aligned} L/D &= L/D \text{ ratio} \\ C &= \text{lb fuel/B.HP/hr} \\ \eta &= \text{propeller efficiency} \end{aligned}\right\} \text{at speed } V\left(mph\right)$$

$$\begin{aligned} W_0 &= \text{initial gross weight, lb} \\ W_1 &= \text{final gross weight, lb} \end{aligned}$$

Example: For the same airplane in Problem I, find the maximum endurance (hours) of flight, if the airplane carries 50 gal of fuel.

Solution: Referring to the point of tangency of the horizontal line, *BC*, Fig. 10-2, it is noted that the point of minimum power required determines the velocity for maximum endurance to be 63.8 mph. Then, in order to determine the L/D of the complete airplane at this velocity, it is only necessary to refer to Table IV and find the total drag at that speed. The wing drag of 199.3 lb plus the parasite drag of 57.3 lb gives a total drag of 256.6 lb. The weight of the airplane is 3000 lb, therefore, for level flight, the L/D of the complete airplane is $3000/256.6 = 11.67$. It may also be noted from the data of power available, Table V, that for a velocity of 63.8 mph (by approximate interpolation) the efficiency, η, of the fixed pitch propeller is 0.64.

The specific fuel consumption is 0.55 lb/B.HP/hr and the total initial gross weight, W_0, minus 50 gal of fuel at 6 lb/gal is

$$3000 - 300 = 2700 \text{ lb} = W_1 \text{ (final gross weight)}$$

Substituting the above values in the Brequet formula.

$$\text{Endurance (hr)} = 750 \times \frac{\sqrt{3000}}{63.8}\left(\frac{3000}{256.6}\right)\frac{0.64}{0.55}\left[\frac{1}{\sqrt{2700}} - \frac{1}{\sqrt{3000}}\right]$$

$$= 750 \times 0.861 \times 11.67 \times 1.164 \times 0.00096$$

$$\text{Endurance (max)} = 8.42 \text{ hr}$$

In the above example, the specific fuel consumption was given as 0.55 lb B.HP/hr. Although this is usually the value assumed in a problem of this sort, the range of fuel consumption may be from 0.45 to 0.55 lb/B.HP/hr, depending on the particular engine used and its condition. Unless the manufacturer's specifications are available, 0.55 is considered to be accurate enough for practical purposes.

If an airplane is operated at any speed beyond that for minimum power required, the endurance will be considerably reduced. This may seem strange, but it must be kept in mind that endurance does not depend on airspeed but only on the speed for best economy. As a rule, an airplane will have poorest endurance at its maximum speed. Even though the specific fuel consumption should remain unchanged (which is untrue), the endurance, which is a function of velocity and drag, would be reduced with an increase in velocity and a corresponding drag increase.

Example: Find the endurance of the same airplane (in the preceding problem) if operated at maximum speed (144 mph).

Solution: From Table IV, at 144 mph, the wing drag of 175 lb plus parasite drag of 308 lb (by approximate interpolation) gives a total drag of 483 lb. Then the total L/D ratio is $3000/483 = 6.22$.

The propeller efficiency at the maximum speed of 144 mph was calculcated to be 0.83. (See Table V.) If the specific fuel consumption is assumed to be 0.55 lb/B.HP/hr then the endurance is

Endurance (at top speed) =

$$750 \times \frac{\sqrt{3000}}{144}\left(\frac{3000}{483}\right)\left(\frac{0.83}{0.55}\right)\left[\frac{1}{\sqrt{2700}} - \frac{1}{\sqrt{3000}}\right]$$

$$= 750 \times 0.381 \times 6.22 \times 1.51 \times 0.00096$$

$$= 2.58 \text{ hr}$$

Under normal flight conditions, an airplane will be operated neither at the speed for least power required nor at its maximum speed. The usual cruising speed of an airplane is determined by considering the engine operating at 60–75% B.HP (rated). If the engine in the above example were to be operated at 60 percent of its rated power, the cruising speed would be 129 mph at sea level. Therefore, the endurance for this speed must be calculated, as above, to find the normal endurance of the airplane.

Range. The *range* of an airplane is a function of the total L/D ratio, the efficiency of the propeller at the operating speed, and the specific fuel consumption. As a rule, the speed for maximum range

is greater than that for maximum endurance, since the object of the flight is to cover the greatest distance for a given supply of gasoline. It follows that the airplane should fly at the angle of attack of maximum efficiency, or, more specifically, the angle of attack of best L/D_{total}. This differs from the angle of attack of minimum $C_D/C_L^{3/2}$ in that, at this angle, the least power is required to sustain flight, whereas the angle of attack for maximum L/D_{total} will require a higher velocity and at the same time will be the point for minimum fuel consumption for a given distance. In Fig. 10-2, a line, FE, is drawn from the origin of the graph (0 horsepower and 0 velocity) tangent to the horsepower-required curve, the point of tangency representing the minimum ratio of horsepower to velocity. For this particular airplane, the point of tangency determines the maximum L/D_{total} of the airplane to be at the airspeed of 78 mph and 47 HP required. Referring to Table IV, at an approximate speed of 78 mph, the airplane is flying level at an angle of attack of 8°. So, for the maximum range, the airplane should be flown at an angle of attack of 8°, and an airspeed of 78 mph. Recalling statements in Chapter I concerning the angle of attack of best L/D compared with angle of attack of best $C_L^{3/2}/C_D$, it was brought out that the angle of attack for best L/D was less than the angle of attack for best $C_L^{3/2}/C_D$, because one represented the angle of attack for best efficiency and the other, the angle for least power required. This point was proved in the section on Endurance where the angle of best $C_L^{3/2}/C_{D\text{total}}$ was found to be 12° and for best L/D_{total}, the angle of attack is noted to be a smaller angle, namely, 8°.

One of the more frequently used expressions for range is Brequet's formula (N.A.C.A. Technical Report No. 234):

$$\text{Range (miles)} = 864 \left(\frac{L}{D}\right) \frac{\eta}{C} \log_{10} \left(\frac{W_0}{W_1}\right)$$

where
$$\begin{aligned} L/D &= L/D_{\text{total}} \\ C &= \text{lb fuel/B.HP/hr} \\ \eta &= \text{propeller efficiency} \\ W_0 &= \text{initial gross weight, lb} \\ W_1 &= \text{final gross weight, lb} \end{aligned}$$

Example: Find the maximum range of the same airplane as in the preceding example.

Solution: The line, FE, drawn tangent to HP required curve, Fig. 10-2, determines best L/D at 78 mph and HP required is 47. Referring to Table IV for velocity of 78 mph the wing drag is 150 lb and the parasite drag is 80 lb (approximately). Then, if airplane weighs 3000 lb, the L/D is $3000/230 = 13.05$. For 78 mph read propeller efficiency to be 72 percent (approximately), Table V. Assuming the specific fuel consumption to be 0.55 lb/B.HP/hr and the fuel capacity as 50 gal, the maximum range is

$$\text{Range (miles)} = 864 \times \left(\frac{3000}{230}\right) \times \frac{0.72}{0.55} \times \log_{10}\left(\frac{3000}{2700}\right)$$

$$= 864 \times 13.05 \times 1.31 \times 0.0453$$

$$= 670 \text{ mi}$$

Example: Find the range of the airplane when cruising at 60 percent of rated power.

Solution: $0.60 \times 220 = 132$ HP at cruising. Read 130 mph at 132 HP, Fig. 10-2. Referring to Table IV for velocity of 130 mph, wing drag is 154 lb and parasite drag is 246 (approx.). Airplane weighs 3000 lb. Therefore, L/D is $3000/400 = 7.5$. Read propeller efficiency to be 0.815 for velocity of 130 mph. When fuel capacity is 50 gal and specific fuel consumption is 0.55 lb/B.HP/hr., the range will be

$$\text{Range (miles)} = 864 \times \left(\frac{3000}{400}\right) \times \frac{0.815}{0.55} \times \log_{10}\left(\frac{3000}{2700}\right)$$

$$= 864 \times 7.5 \times 1.484 \times 0.0453$$

$$= 436 \text{ mi}$$

It is not difficult to see from the above examples why airplane owners sometimes find there is a considerable difference in the range they expected and the actual range attained. An airplane flown at a velocity greater or less than that for best L/D_{total} will realize a decided decrease in the maximum design range.

APPENDIX

*AERONAUTICAL NOMENCLATURE**

accelerometer—An instrument that measures the accelerations of an aircraft in a defined direction.

acrobatics—Evolutions voluntarily performed with an aircraft, other than those required for normal flight.

adjustable propeller—See Propeller, Adjustable.

aerodynamic center, wing section—A point located on or near the chord of the mean line approximately one quarter of the chord length aft of the leading edge and about which the moment coefficient is practically constant.

aerodynamics—The branch of dynamics that treats of the motion of air and other gaseous fluids and of the forces acting on solids in motion relative to such fluids.

aerostatics—The science that treats of the equilibrium of gaseous fluids and of bodies immersed in them.

aileron—A hinged or movable portion of an airplane wing, the primary function of which is to impress a rolling motion on the airplane. It is usually part of the trailing edge of a wing.

external aileron—A separate airfoil mounted clear of the wing surfaces of an airplane but usually attached to them and deflected for lateral control.

Frise aileron—An aileron having the nose portion projecting ahead of the hinge axis, the lower surface being in line with the lower surface of the wing. When the trailing edge of the aileron is raised, the nose portion protrudes below the lower surface of the wing, increasing the drag.

slotted aileron—An aileron having a nose and axis arrangement somewhat similar to a Frise aileron but having a smooth air passage between the nose portion of the aileron and the wing for the purpose of maintaining a smooth airflow over the upper surface of the aileron when its trailing edge is deflected downward.

upper-surface aileron—A split flap forming the rear upper surface of a wing, deflected for lateral control.

aileron linkage arrangement:

differential aileron linkage arrangement—Ailerons so interconnected that a given movement of the control stick results in the upward displacement of one aileron being greater than the downward displacement of the other.

floating aileron linkage arrangement—Ailerons so linked together and to the control stick as to "float" freely in the air stream except when displaced by the lateral motion of the control stick.

aircraft—Any weight-carrying device designed to be supported by the air, either by buoyancy or by dynamic action.

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* **Based on N.A.C.A. Technical Report 474.**

airfoil—Any surface, such as an airplane wing, aileron, or rudder, designed to obtain reaction from the air through which it moves.

airfoil profile—The outline of an airfoil section.

airfoil section—A cross section of an airfoil parallel to the plane of symmetry or to a specified reference plane.

airplane—A mechanically driven fixed-wing aircraft, heavier than air, which is supported by the dynamic reaction of the air against its wings.

canard airplane—A type of airplane having the horizontal stabilizing and control surfaces in front of the main supporting surfaces.

pusher airplane—An airplane with the propeller or propellers aft of the main supporting surfaces.

tailless airplane—An airplane in which the devices used to obtain stability and control are incorporated in the wing.

tractor airplane—An airplane with the propeller or propellers forward of the main supporting surfaces.

air speed—The speed of an aircraft relative to the air.

air-speed head—An instrument which, in combination with a gage, is used to measure the speed of an aircraft relative to the air. It usually consists of a pitot-static tube or a pitot-venturi tube.

airway—An air route along which aids to air navigation, such as landing fields, beacon lights, radio direction-finding facilities, intermediate fields, etc., are maintained.

airworthiness—The quality of an aircraft denoting its fitness and safety for operation in the air under normal flying conditions.

altimeter—An instrument that measures the elevation of an aircraft above a given datum plane.

altitude:

absolute altitude—The height of an aircraft above the earth.

critical altitude—The maximum altitude at which a supercharger can maintain a pressure in the intake manifold of an engine equal to that existing during normal operation at rated power and speed at sea level.

density altitude—The altitude corresponding to a given density in a standard atmosphere.

pressure altitude—(1) The altitude corresponding to a given pressure in a standard atmosphere. (2) The altitude at which the gas bags of an airship become full.

amphibian—An airplane designed to rise from and alight on either land or water.

angle:

aileron angle—The angular displacement of an aileron from its neutral position. It is positive when the trailing edge of the aileron is below the neutral position.

blade angle—The acute angle between the chord of a section of a propeller, or of a rotary wing system, and a plane perpendicular to the axis of rotation.

coning angle—The average angle between the span axis of a blade or wing of a rotary wing system and a plane perpendicular to the axis of rotation.

dihedral angle—The acute angle between a line perpendicular to the plane of symmetry and the projection of the wing axis on a plane

perpendicular to the longitudinal axis of the airplane. If the wing axis is not approximately a straight line, the angle is measured from the projection of a line joining the intersection of the wing axis with the plane of symmetry and the aerodynamic center of the half-wing on either side of the plane of symmetry.

downwash angle—The angle through which an air stream is deflected by any lifting surface. It is measured in a plane parallel to the plane of symmetry.

drift angle—The horizontal angle between the longitudinal axis of an aircraft and its path relative to the ground.

effective helix angle—The angle of the helix described by a particular point on a propeller blade as the airplane moves forward through air otherwise undisturbed.

elevator angle—The angular displacement of the elevator from its neutral position. It is positive when the trailing edge of the elevator is below the neutral position.

flapping angle—The difference between the coning angle and the instantaneous angle of the span axis of a blade of a rotary wing system relative to the plane perpendicular to the axis of rotation.

flight-path angle—The angle between the flight path of the aircraft and the horizontal.

gliding angle—The angle between the flight path during a glide and a horizontal axis fixed relative to the air.

landing angle—The acute angle between the wing chord and the horizontal when the airplane is resting on level ground in its normal position; also called "ground angle."

minimum gliding angle—The acute angle between the horizontal and the most nearly horizontal path along which an airplane can descend steadily in still air when the propeller is producing no thrust.

rudder angle—The acute angle between the rudder and the plane of symmetry of the aircraft. It is positive when the trailing edge has moved to the left with reference to the normal position of the pilot.

trim angle—The angle between the horizontal and the longitudinal base line of a seaplane float or flying-boat hull. It is positive when the bow is higher than the stern.

zero-lift angle—The angle of attack of an airfoil when its lift is zero.

angle of attack—The acute angle between a reference line in a body and the line of the relative wind direction projected on a plane containing the reference line and parallel to the plane of symmetry.

absolute angle of attack—The angle of attack of an airfoil, measured from the attitude of zero lift.

critical angle of attack—The angle of attack at which the flow about an airfoil changes abruptly as shown by corresponding abrupt changes in the lift and drag.

effective angle of attack—See Angle of Attack for Infinite Aspect Ratio.

induced angle of attack—The difference between the actual angle of attack and the angle of attack for infinite aspect ratio of an airfoil for the same lift coefficient.

angle of attack for infinite aspect ratio—The angle of attack at which

the lower surface; and "mean camber" to the mean line of the section. Camber is positive when the departure is upward, and negative when it is downward.

center of pressure of an airfoil—The point in the chord of an airfoil, prolonged if necessary, which is at the intersection of the chord and the line of action of the resultant air force.

center-of-pressure coefficient—The ratio of the distance of the center of pressure from the leading edge to the chord length.

center section—The central panel of a wing; in the case of a continuous wing or any wing having no central panel, the limits of the center section are arbitrarily defined by the location of points of attachment to the cabane struts or fuselage.

chandelle—An abrupt climbing turn to approximately a stall in which the momentum of the airplane is used to obtain a higher rate of climb than would be possible in unaccelerated flight. The purpose of this maneuver is to gain altitude at the same time that the direction of flight is changed.

chord—An arbitrary datum line from which the ordinates and angles of an airfoil are measured. It is usually the straight line tangent to the lower surface at two points, the straight line joining the ends of the mean line or the straight line between the leading and trailing edges.

chord, mean aerodynamic—The chord of an imaginary airfoil which would have force vectors throughout the flight range identical with those of the actual wing or wings.

chord, mean, of a wing—The quotient obtained by dividing the wing area by the span.

control column—A lever having a rotatable wheel mounted at its upper end for operating the longitudinal and lateral control surfaces of an airplane. This type of control is called "wheel control."

control, servo—A control devised to reinforce the pilot's effort by an aerodynamic or mechanical relay.

controllability—The quality of an aircraft that determines the ease of operating its controls and/or the effectiveness of displacement of the controls in producing change in its attitude in flight.

controls—A general term applied to the means provided to enable the pilot to control the speed, direction of flight, attitude, power, etc., of an aircraft.

control stick—The vertical lever by means of which the longitudinal and lateral control surfaces of an airplane are operated. The elevator is operated by a fore-and-aft movement of the stick; the ailerons, by a side-to-side movement.

control surface—A movable airfoil designed to be rotated or otherwise moved by the pilot in order to change the attitude of the aircraft.

cowling—A removable covering.

cockpit cowling—A metal or plywood cowling placed around a cockpit.

engine cowling—A removable covering placed around all or part of an airplane engine.

N.A.C.A. cowling—A cowling enclosing a radial air-cooled engine, consisting of a hood, or ring, and a portion of the body behind the

engine so arranged that the cooling air smoothly enters the hood at the front and leaves through a smooth annular slot between the body and the rear of the hood; the whole forming a relatively low-drag body with a passage through a portion of it for the cooling air.

directional gyro—A gyroscopic instrument for indicating direction, containing a free gyroscope which holds its position in azimuth and thus indicates angular deviation from the course.

dive—A steep descent, with or without power, in which the airspeed is greater than the maximum speed in horizontal flight.

downwash—The air deflected perpendicular to the direction of motion of an airfoil.

drag—The component of the total air force on a body parallel to the relative wind.

induced drag—That part of the drag induced by the lift.

parasite drag—That portion of the drag of an aircraft exclusive of the induced drag of the wings.

profile drag, effective—The difference between the total wing drag and the induced drag of wing with the same geometric aspect ratio but elliptically loaded.

dynamic pressure—The product $^1/_2\rho V^2$, where ρ is the density of the air and V is the relative speed of the air.

empennage—See Tail, Airplane.

fairing—An auxiliary member or structure whose primary function is to reduce the drag of the part to which it is fitted.

feather—In rotary wing systems, to periodically increase and decrease the incidence of a blade or wing by oscillating the blade or wing about its span axis.

fin—A fixed or adjustable airfoil, attached to an aircraft approximately parallel to the plane of symmetry, to afford directional stability; for example, tail fin, skid fin, etc.

fineness ratio—The ratio of the length to the maximum diameter of a streamline body, as an airship hull.

fitting—A generic term for any small part used in the structure of an airplane or airship. If without qualification, a metal part is usually understood. It may refer to other parts, such as fabric fittings.

flap—A hinged or pivoted airfoil forming the rear portion of an airfoil, used to vary the effective camber.

split flap—A hinged plate forming the rear lower portion may be deflected downward to give increased lift and drag; the upper portion may be raised over a portion of the wing for the purpose of lateral control.

fuselage—The body, of approximately streamline form, to which the wings and tail unit of an airplane are attached.

monocoque fuselage—A fuselage construction which relies on the strength of the skin or shell to carry either the shear or the load due to bending moments. Monocoques may be divided into three classes (reinforced shell, semimonocoque, and monocoque), and different portions of the same fuselage may belong to any one of these classes. The reinforced shell has the skin reinforced by a complete framework of structural members. The semimonocoque has the skin reinforced by longerons and vertical

bulkheads, but has no diagonal web members. The monocoque has, as its only reinforcement, vertical bulkheads formed of structural members.

ground loop—An uncontrollable violent turn of an airplane while taxiing, or during the landing or take-off run.

helicopter—A type of rotor plane whose support in the air is normally derived from airfoils mechanically rotated about an approximately vertical axis.

horsepower of an engine, rated—The average horsepower developed by a given type of engine at the rated speed when operating at full throttle, or at a specified altitude or manifold pressure.

Immelman turn, normal—A maneuver made by completing the first half of a normal loop; from the inverted position at the top of the loop, half-rolling the airplane to the level position, thus obtaining a 180° change in direction simultaneously with a gain in altitude.

impact pressure—The pressure acting at the forward stagnation point of a body, such as a pitot tube placed in an air current. Impact pressure may be measured from an arbitrary datum pressure.

inclinometer—An instrument that measures the attitude of an aircraft with respect to the horizontal.

instability, spiral—A type of instability, inherent in certain airplanes, which becomes evident when the airplane assumes too great a bank and sideslips; the bank continues to increase and the radius of the turn to decrease.

interceptor—A lateral-control device consisting of a small plate placed just back of a wing slot to spoil the effect of the slot at high angles of attack.

landing—The act of terminating flight in which the aircraft is made to descend, lose flying speed, establish contact with the ground, and finally come to rest.

glide landing—A landing in which a steady glide is maintained to the landing surface without the usual leveling-off before contact.

level landing (stress analysis)—A loading condition for the fuselage and landing gear, representing a two-point landing with the fuselage horizontal.

normal (or three-point) landing—A landing in which a path tangential to the landing surface and the loss in flying speed are attained at approximately the instant of contact.

pancake landing—A landing in which the leveling-off process is carried out several feet above the ground, as a result of which the airplane settles rapidly on a steep flight path in a normal attitude.

three-point landing (stress analysis)—A loading condition for the fuselage and landing gear, representing landing with the wheels and tall skid touching the ground simultaneously (cf. level landing).

landing gear—The understructure which supports the weight of an aircraft when in contact with the land or water and which usually contains a mechanism for reducing the shock of landing. Also called "undercarriage."

retractable landing gear—A type of landing gear which may be with-

drawn into the body or wings of an airplane while it is in flight, in order to reduce the parasite drag.

lift/drag ratio—The ratio of the lift to the drag of any body.

load:

basic load (*stress analysis*)—The load on a structural member or part in any condition of static equilibrium of an airplane. When a specific basic load is meant, the particular condition of equilibrium must be indicated in the context.

design load (*stress analysis*)—A specified load below which a structural member or part should not fail. It is the probable maximum applied load multiplied by the factor of safety. Also, in many cases, an appropriate basic load multiplied by a design load factor.

full load—Weight empty plus useful load; also called "gross weight."

normal load (*stress analysis*)—The load on that part of a wing assumed to be unaffected by tip losses or similar corrections. In any given case, it may be a basic design, gross, net, or ultimate load, depending on the context.

pay load—That part of the useful load from which revenue is derived, *viz*, passengers and freight.

ultimate load (*stress analysis*)—The load that causes destructive failure in a member during a strength test, or the load that, according to computations, should cause destructive failure in the member.

useful load—The crew and passengers, oil and fuel, ballast other than emergency, ordnance, and portable equipment.

load factor (*stress analysis*)—The ratio of two loads (the second being a basic load) that have the same relative distribution. The first load may be the load applied during some special maneuver, the maximum probable load on the airplane or part, the design load, or the ultimate load. Whenever a load factor is mentioned, the context should indicate clearly what load is being compared with the basic load. If the context does not so indicate, the load factor is usually the ratio of the design load to the weight of the airplane.

loading:

power loading—The gross weight of an airplane divided by the rated horsepower of the engine computed for air of standard density, unless otherwise stated.

span loading—The ratio of the weight of an airplane to its equivalent monoplane span.

unsymmetrical loading (*stress analysis*)—A design loading condition for the wings and connecting members, representing the conditions as in a roll.

wing loading—The gross weight of an airplane divided by the wing area.

longeron—A principal longitudinal member of the framing of an airplane fuselage or nacelle, usually continuous across a number of points of support.

loop—A maneuver executed in such a manner that the airplane follows a closed curve approximately in a vertical plane.

inverted normal loop—A loop starting from inverted flight and pass-

ing successively through a dive, normal flight, climb, and back to inverted flight.

inverted outside loop—An outside loop starting from inverted flight and passing successively through a climb, normal flight, dive and back to inverted flight.

normal loop—A loop starting from normal flight and passing successively through a climb, inverted flight, dive, and back to normal flight.

outside loop—A loop starting from normal flight and passing successively through a dive, inverted flight, climb, and back to normal flight, the pilot being on the outside of the flight path.

maneuverability—That quality in an aircraft which determines the rate at which its attitude and direction of flight can be changed.

mean line (of an airfoil profile)—An intermediate line between the upper and lower contours of the profile.

monoplane—An airplane with but one main supporting surface sometimes divided into two parts by the fuselage.

high-wing monoplane—A monoplane in which the wing is located at, or near, the top of the fuselage.

low-wing monoplane—A monoplane in which the wing is located at, or near, the bottom of the fuselage.

midwing monoplane—A monoplane in which the wing is located approximately midway between the top and bottom of the fuselage.

parasol monoplane—A monoplane in which the wing is above the fuselage.

nacelle—An enclosed shelter for personnel or for a power plant. A nacelle is usually shorter than a fuselage, and does not carry the tail unit.

noseheavy—The condition of an airplane in which the nose tends to sink when the longitudinal control is released in any given attitude of normal flight (see tailheavy).

ornithopter—A form of aircraft heavier than air, deriving its chief support and propelling force from flapping wings.

oscillation:

phugoid oscillation—A long-period oscillation characteristic of the disturbed longitudinal motion of an aircraft.

stable oscillation—An oscillation whose amplitude does not increase.

unstable oscillation—An oscillation whose amplitude increases continuously until an attitude is reached from which there is no tendency to return toward the original attitude, the motion becoming a steady divergence.

pilot—One who operates the controls of an aircraft in flight.

pitch—An angular displacement about an axis parallel to the lateral axis of an aircraft.

pitch of propeller:

effective pitch—The distance an aircraft advances along its flight path for one revolution of the propeller.

geometrical pitch—The distance an element of a propeller would advance in one revolution if it were moving along a helix having an angle equal to its blade angle.

zero-thrust pitch—The distance a propeller would have to advance

in one revolution to give no thrust. Also called "experimental mean pitch."

pitch ratio (propeller)—The ratio of the pitch to the diameter.

pitot-static tube—A parallel or coaxial combination of a pitot and a static tube. The difference between the impact pressure and the static pressure is a function of the velocity of flow past the tube.

profile thickness—The maximum distance between the upper and lower contours of an airfoil, measured perpendicularly to the mean line of the profile.

propeller—Any device for propelling a craft through a fluid, such as water or air; especially a device having blades which, when mounted on a power-driven shaft, produce a thrust by their action on the fluid.

adjustable propeller—A propeller whose blades are so attached to the hub that the pitch may be changed while the propeller is at rest.

automatic propeller—A propeller whose blades are attached to a mechanism that automatically sets them at their optimum pitch for various flight conditions.

controllable propeller—A propeller whose blades are so mounted that the pitch may be changed while the propeller is rotating.

geared propeller—A propeller driven through gearing, generally at some speed other than the engine speed.

pusher propeller—A propeller mounted on the rear end of the engine or propeller shaft.

tractor propeller—A propeller mounted on the forward end of the engine or propeller shaft.

propeller efficiency—The ratio of the thrust power to the input power of a propeller.

propeller rake—The mean angle which the line joining the centroids of the sections of a propeller blade makes with a plane perpendicular to the axis.

propeller root—That part of the propeller blade near the hub.

propeller thrust—The component of the total air force on the propeller which is parallel to the direction of advance.

propeller thrust, effective—The net driving force developed by a propeller when mounted on an aircraft, *i.e.*, the actual thrust exerted by the propeller, as mounted on an airplane, minus any increase in the resistance of the airplane due to the action of the propeller.

propeller thrust, static—The thrust developed by a propeller when rotating without translation.

propulsive efficiency—The ratio of the product of the effective thrust and flight speed to the actual power input into the propeller as mounted on the airplane.

range, maximum—The maximum distance a given aircraft can cover under given conditions, by flying at the economical speed and altitude at all stages of the flight.

range at maximum speed—The maximum distance a given aircraft can fly at full speed at the altitude for maximum speed under given conditions.

Reynolds Number—A nondimensional coefficient used as a measure of the dynamic scale of a flow. Its usual form is the fraction $\rho(Vl/\nu)$ in which ρ is the density of the fluid, V is the velocity of the fluid, l is

a linear dimension of a body in the fluid, and ν is the coefficient of viscosity of the fluid (see scale effect).

roll—A maneuver in which a complete revolution about the longitudinal axis is made, the horizontal direction of flight being approximately maintained.

aileron roll—A roll in which the motion is largely maintained by forces arising from the displacement of the aileron.

outside roll—A roll executed while flying in the negative angle-of-attack range.

snap roll—A roll executed by a quick movement of the controls, in which the motion is largely maintained by autorotational couples on the wings.

scale effect—The change in any force coefficient, such as the drag coefficient, due to a change in the value of Reynolds Number.

skidding—Sliding sidewise away from the center of curvature when turning. It is caused by banking insufficiently, and is the opposite of sideslipping.

skin friction—The tangential component of the fluid force at a point on a surface.

slat—A movable auxiliary airfoil, attached to the leading edge of a wing, which, when closed, falls within the original contour of the main wing and which, when opened, forms a slot.

slip—The difference between the geometrical pitch and the effective pitch of a propeller. Slip may be expressed as a percentage of the mean geometrical pitch, or as a linear dimension.

slip function—The ratio of the speed of advance through the undisturbed air to the product of the propeller diameter and the number of revolutions per unit time; *i.e.*, V/nD.

slipstream—The current of air driven astern by a propeller.

slot—The nozzle-shaped passage through a wing whose primary object is to improve the flow conditions at high angles of attack. It is usually near the leading edge and formed by a main and an auxiliary airfoil, or slat.

solidity—The ratio of the total blade area of a rotor to the area of the disk swept by the blades.

span—The maximum distance, measured parallel to the lateral axis, from tip to tip of an airfoil, of an airplane wing inclusive of ailerons, or of a stabilizer inclusive of elevator.

effective span—The true span of a wing less corrections for tip loss.

spin—A maneuver in which an airplane descends along a helical path of large pitch and small radius while flying at a mean angle of attack greater than the angle of attack at maximum lift (see spiral).

flat spin—A spin in which the longitudinal axis is less than 45° from the horizontal.

inverted spin—A maneuver having the characteristics of a normal spin except that the airplane is in an inverted attitude.

normal spin—A spin which is continued by reason of the voluntary position of the control surfaces, recovery from which can be effected within two turns by neutralizing or reversing all the controls. Sometimes called "controlled spin."

uncontrolled spin—A spin in which the controls are of little or no use in effecting a recovery.

spiral—A maneuver in which an airplane descends in a helix of small pitch and large radius, the angle of attack being within the normal range of flight angles.

split S—A maneuver consisting of a half snap roll followed by a pull-out to normal flight, thus obtaining at 180° change in direction accompanied by a loss of altitude.

spoiler—A small plate arranged to project above the upper surface of a wing to disturb the smooth airflow, with consequent loss of lift and increase of drag.

stability—That property of a body which causes it, when its equilibrium is disturbed, to develop forces or moments tending to restore the original condition.

automatic stability—Stability dependent upon movable control surfaces automatically operated by mechanical means.

directional stability—Stability with reference to disturbances about the normal axis of an aircraft, *i.e.*, disturbances which tend to cause yawing.

dynamic stability—That property of an aircraft which causes it, when its state of steady flight is disturbed, to damp the oscillations set up by the restoring forces and moments and gradually return to its original state.

inherent stability—Stability of an aircraft due solely to the disposition and arrangement of its fixed parts; *i.e.*, that property which causes it, when disturbed, to return to its normal attitude of flight without the use of the controls or the interposition of any mechanical device.

lateral stability—Stability with reference to disturbances about the longitudinal axis; *i.e.*, disturbances involving rolling or sideslipping. The term lateral stability is sometimes used to include both directional and lateral stability, since these cannot be entirely separated in flight.

longitudinal stability—Stability with reference to disturbances in the plane of symmetry, *i.e.*, disturbances involving pitching and variation of the longitudinal and normal velocities.

static stability—That property of an aircraft which causes it, when its state of steady flight is disturbed, to develop forces and moments tending to restore its original condition.

stabilizer (*airplane*)—Any airfoil whose primary function is to increase the stability of an aircraft. It usually refers to the fixed horizontal tail surface of an airplane, as distinguished from the fixed vertical surface.

stagger—A term referring to the longitudinal position of the axes of two wings of an airplane. Stagger of any section is measured by the acute angle between a line joining the wing axes and a line perpendicular to the upper wing chord, both lines lying in a plane parallel to the plane of symmetry. The stagger is positive when the upper wing is in advance of the lower.

stall—The condition of an airfoil or airplane in which it is operating at an angle of attack greater than the angle of attack of maximum lift.

sting—A light rod attached to and extending backward from a body for convenience of mounting when testing in a wind tunnel.

streamline—The path of a particle of a fluid, supposedly continuous, commonly taken relative to a solid body past which the fluid is moving; generally used only of such flows as are not eddying.

streamline form—The form of a body so shaped that the flow about it tends to be a streamline flow.

supercharger—A pump for supplying the engine with a greater weight of air or mixture than would normally be inducted at the prevailing atmospheric pressures.

centrifugal-type supercharger—A high-speed rotary blower equipped with one or more multi-blade impellers which, through centrifugal action, compress the air or mixture in the induction system.

positive-driven-type supercharger—A supercharger driven at a fixed speed ratio from the engine shaft by gears or other positive means.

reciprocating-type supercharger—A positive-displacement reciprocating pump in which the air or mixture is compressed by a piston working in a cylinder.

Roots-type supercharger—A positive-displacement rotary blower consisting of two double-lobed impellers turning in opposite directions on parallel shafts within a housing, the impellers rolling together except for a small clearance, meanwhile alternately trapping incoming air or mixture in the ends of the housing and sweeping it through to the outlet.

vane-type supercharger—A positive-displacement rotary blower having an eccentrically located rotor provided with one or more vanes.

sweepback—The acute angle between a line perpendicular to the plane of symmetry and the plan projection of a reference line in the wing.

tab—An auxiliary airfoil attached to a control surface for the purpose of reducing the control force or trimming the aircraft.

tachometer—An instrument that measures in revolutions per minute the rate at which the crankshaft of an engine turns.

tail, airplane—The rear part of an airplane, usually consisting of a group of stabilizing planes, or fins, to which are attached certain controlling surfaces such as elevators and rudders; also called "empennage."

tailheavy—The condition of an airplane in which the tail tends to sink when the longitudinal control is released in any given attitude of normal flight (see noseheavy).

take-off—The act of beginning flight in which an airplane is accelerated from a state of rest to that of normal flight. In a more restricted sense, the final breaking of contact with the land or water.

taper in plan only—A gradual change (usually a decrease) in the chord length along the wing span from the root to the tip, with the wing sections remaining geometrically similar.

taper in thickness ratio only—A gradual change in the thickness ratio along the wing span with the chord remaining constant.

taxi—To operate an airplane under its own power either on land or on water, except as necessarily involved in take-off or landing.

trim:

(airplane) trim—The attitude with respect to wind axes at which balance occurs in rectilinear flight with free controls.

(*airship*) *trim*—The attitude of the longitudinal axis of an airship with respect to the horizontal.

(*seaplane*) *trim*—The angle with the horizontal surface of the water assumed by the float or hull under given conditions.

Venturi tube (*or Venturi*)—A short tube of varying cross section. The flow through the Venturi causes a pressure drop in the smallest section, the amount of the drop being a function of the velocity of flow.

visibility—The greatest distance at which conspicuous objects can be seen and identified.

weight:

dischargeable (*or consumable*) *weight* (*airship*)—All weight that can be consumed or discharged and still leave the airship in safe operating condition with a specified reserve of fuel, oil, water ballast, and provisions, and with the normal crew.

empty weight—The structure, power plant, and fixed equipment of an aircraft. Included in this fixed equipment are the water in the radiator and cooling system, all essential instruments and furnishings, fixed electric wiring for lighting, heating, etc. In the case of an aerostat, it includes the amount of ballast that must be carried to assist in making a safe landing.

fixed power-plant weight for a given airplane weight—The weight of the power plant and its accessories, exclusive of fuel and oil and their tanks.

gross weight (*airplane*)—The total weight of an airplane when fully loaded (see load, full).

wheel, tail—A wheel used to support the tail of an airplane when on the ground. It may be steerable or nonsteerable, fixed or swiveling.

wind, relative—The velocity of the air with reference to a body in it. It is usually determined from measurements made at such a distance from the body that the disturbing effect of the body upon the air is negligible.

wind tunnel—An apparatus producing an artificial wind or air stream, in which objects are placed for investigating the air flow about them and the aerodynamic forces exerted on them.

wing—A general term applied to the airfoil, or one of the airfoils, designed to develop a major part of the lift of a heavier-than-air craft.

wingheavy, right or left—The condition of an airplane whose right or left wing tends to sink when the lateral control is released in any given attitude of normal flight.

wing-over—A maneuver in which the airplane is put into a climbing turn until nearly stalled, at which point the nose is allowed to fall while continuing the turn, then returned to normal flight from the ensuing dive or glide in a direction approximately 180° from that at the start of the evolution.

wing rib—A chord-wise member of the wing structure of an airplane, used to give the wing section its form and to transmit the load from the fabric to the spars.

compression wing rib—A heavy rib designed to perform the function of an ordinary wing rib and also to act as a strut opposing the pull of the wires in the internal drag truss.

former (*or false*) *wing rib*—An incomplete rib, frequently consisting

only of a strip of wood extending from the leading edge to the front spar, which is used to assist in maintaining the form of the wing where the curvature of the airfoil section is sharpest.

wing section—A cross section of a wing parallel to the plane of symmetry or to a specified reference plane.

wing skid—A skid placed near the wing tip to protect the wing from contact with ground.

wing spar—A principal span-wise member of the wing structure of an airplane.

wing-tip rake—A term referring to the shape of the tip of the wing when the tip edge is sensibly straight in plan but is not parallel to the plane of symmerty. The amount of rake is measured by the acute angle between the straight portion of the wing tip and the plane of symmetry. The rake is positive when the trailing edge is longer than the leading edge.

wire (airplane):

antidrag wire—A wire intended primarily to resist the forces acting forward in the chord direction. It is generally enclosed in the wing.

drag wire—A wire intended primarily to resist the forces acting backward in the chord direction. It is generally enclosed in the wing.

landing wire—A wire or cable which braces the wing against the forces opposite to the normal direction of the lift.

lift wire—A wire or cable which braces the wings against the lift force; sometimes called "flying wire."

stagger wire—A wire connecting the upper and lower wings of an airplane and lying in a plane substantially parallel to the plane of symmetry; also called "incidence wire."

antiflutter wire—A wire in the plane of the outer cover for local reinforcement and for reducing flutter due to variations in air pressure or propeller wash.

chord wire—A wire joining the vertices of a main transverse frame.

fairing wire—A wire provided as a point of attachment for the outer cover to maintain the contour lines of the envelope of an airship.

yaw—An angular displacement about an axis parallel to the normal axis of an aircraft.

zero-lift angle—See Angle, Zero-Lift.

zoom—To climb for a short time at an angle greater than the normal climbing angle, the airplane being carried upward at the expense of kinetic energy.

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